Contents

1 Symbolic Calculations **9**

2 Numerical Evaluations **15**
2.1 Taylor Polynomial Approximation 15
2.2 Power Series 17
2.3 Alternating Series 17
2.4 How to evaluate $\sin(x)$ 18
2.5 How to evaluate π 19
2.6 How to evaluate e^x 20
2.7 How to evaluate $\log(x)$ 21
2.8 How to evaluate $\tan^{-1}(x)$ 22
2.9 How to evaluate $\sqrt{x}$ 23

3 Interpolations in 1D **25**
3.1 Polynomial Interpolation 25
3.1.1 Lagrange Form 28
3.1.2 Newton Form 29
3.1.3 Divided Differences 29
3.1.4 Example : Smooth Data 35
3.1.5 Example : Non-smooth Data 36
3.2 Stable Interpolation #1: Cubic Spline Interpolation 37
3.3 Stable Interpolation #2 : ENO Interpolation 44
3.4 Stable Reconstruction : Least Square Fitting 45

4 Interpolations in Multi-Dimensions **49**
4.1 Polynomial Interpolation on Uniform Grids 49
4.2 RBF Interpolation on Scattered Data 50

5 Numerical Linear Algebra : Basic Theory **57**
5.1 Gaussian Elimination Method : LU Factorization 57
5.2 Gaussian-Elimination with Pivoting : PA=LU 59

5.3 Gram-Schmidt Orthogonalization : QR Factorization 62
5.4 Converting Linear System into Symmetric Positive Definite Linear System . 65
5.5 Steepest Descent 65
5.6 Conjugate Gradient 68
5.7 Solving General Linear System : CGNR 70
5.8 Solving General Linear System : Bi-CG 70
5.9 Power Method 71
5.10 QR Method 73
5.10.1 Power Iteration and Deflated Matrix 73
5.10.2 Orthogonal Iteration 75
5.10.3 QR Method 76
5.11 Jacobi Method 78
5.12 Sparse Matrices 81

6 Numerical Linear Algebra : Krylov Space Theory 83
6.1 Krylov Space 84
6.2 Symmetric Positive Definite Matrix : A-Orthogonal Basis 85
6.3 Symmetric-Positive-Definite Matrix : Conjugate Gradient Method 86
6.4 General Nonsingular Matrix : Bi-Orthogonal Bases 89
6.5 General Nonsingular Matrix : Bi-Conjugate Gradient Method 90
6.6 General Nonsingular Matrix: Conjugate Gradient Square Method 93

7 Scientific Visualization 97
7.1 Display System 97
7.2 Drawing Line Segments on the Display 98
7.3 Drawing Triangles on the Display 99
7.4 Drawing Curves on the Display 100
7.5 Drawing 3D objects on the Display 101
7.6 Triangulating 3D Surfaces 102
7.7 Giving Shade Colors to 3D objects 105
7.8 Rotating 3D objects 106
7.8.1 Matrix representation 106
7.8.2 Quaternion Algebra 108

8 Approximating Ordinaray Differential Equations 113
8.1 Mathematical Modeling by Differential Equations 113
8.2 Approximating Derivatives 114
8.3 Basic Methods Approximating Ordinary Differential Equations 116
8.4 Analysis of the Ordinary Differential Equation 119
8.5 Convergence of Forward Euler Method 121
8.6 Convergence of Leapfrog Method 122

8.7 Linear Stability Domain . . . 124
8.8 Example : Lorentz Equation . . . 126
8.8.1 Linear Stability Analysis of the Fixed Points . . . 126
8.8.2 Numerical Simulation of Lorenz Equation . . . 129

9 Approximating Poisson Equation 131
9.0.3 Finite Difference Method . . . 133
9.0.4 Consistency error of the Five-Point Scheme . . . 136
9.0.5 Convergence error of the Five-Point Scheme . . . 137
9.0.6 Example . . . 140

10 Rigid Body Animation 143
10.1 Euler Equations . . . 145
10.2 Collision Mechanics . . . 147
10.2.1 Two Typical Impulses . . . 149
10.2.2 Kinetic Energy Dissipation . . . 150
10.2.3 Coulomb's Friction Law . . . 151
10.2.4 Non-penetration Condition . . . 152
10.2.5 Permissible Region of the Impulse . . . 152
10.2.6 An Algebraic Formula calculating the Impulse . . . 152

11 Computational Fluid Dynamics 157
11.1 Navier-Stokes Equations . . . 157
11.2 Navier-Stokes Equations with Constant Density and Viscosity . . . 159
11.3 Discretizing Time Derivative : Projection Method . . . 160
11.4 Discretizing the Spatial Derivatives . . . 161
11.5 Implementation of the Basic Fluid Solver . . . 164

12 Optimizations 171
12.1 Multi-dimensional Calculus . . . 171
12.2 Gradient Descent Method . . . 175
12.3 Nonlinear Conjugate Gradient Method . . . 178
12.4 Linear Programming . . . 179

13 *Appendices* 187
13.1 Appendix A : Gerchgorin's Circle Theorem . . . 187
13.2 Appendix B : Shur Polynomial Theory . . . 190
13.3 Appendix C : M-matrix Theory . . . 192
13.4 Appendix E : Polar and Spherical Coordinates . . . 193
13.5 Appendix F : Programming in C++ . . . 194
13.5.1 "Hello" Program . . . 195
13.5.2 Basic Data . . . 196

13.5.3 Function 197
13.5.4 Logical Statement 198
13.5.5 Loop Statement 199
13.5.6 Memory Storage 200
13.5.7 Pointer Variables 202
13.5.8 Reference Variables 205
13.5.9 Header and Source Files 206
13.5.10 Class 208
13.5.11 Operator Overloading 210
13.5.12 Using the Library File 212
13.5.13 Project : Mandelbrot Set 215