Motion of a ring structure of coherent vortices on a sphere

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An important aspect to be considered in the study of planetary atmospheric flows is that the fluid is confined on the surface of rotating sphere. As a simple mathematical model, we consider incompressible and inviscid flows on the sphere with two point vortices that are fixed at the north and the south poles of the sphere. In this model, interaction between coherent vortex structures plays an key role since vorticity is invariant along the path of a fluid particle. The pole vortices could generate an outer flow that taken into account the local contribution of rotation of the sphere locally. In the present talk, I would like to give a brief survey of recent research on the motion of coherent vortex structures such as vortex sheets and point vortices on the sphere with pole vortices.

First, we consider the evolution of a vortex sheet, which is a discontinuous surface of the velocity field in incompressible and inviscid flow. The vortex sheet is a mathematical idealization of shear flows that are often observed in atmospheric flows. A flat vortex sheet is linearly unstable, which is known as the Kelvin-Helmholtz instability. Numerical computation reveals that the vortex sheet forms a structure consisting of tightly winding vortex spirals that are aligned along a line of latitude [1]. Thus the Kelvin-Helmholtz instability leads to the generation of a ring structure of coherent vortex-spirals. Next questions are how the latitudinal ring configuration of the vortex spirals evolves when it is unstable. Approximating the tightly winding spirals with identical point vortices, we obtain the polygonal ring structure which is referred to as the *N-ring*. With the point-vortex model, we show that that the unstable N-ring exhibits a chaotic behavior [3, 5, 6].

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