Electrical impedance tomography using equipotential line method

(A new approach to Inverse Conductivity Problem)

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Physical Background

(Forward) Conductivity Problem

$$\textit{For given } \sigma(x),\ x \in \Omega,$$

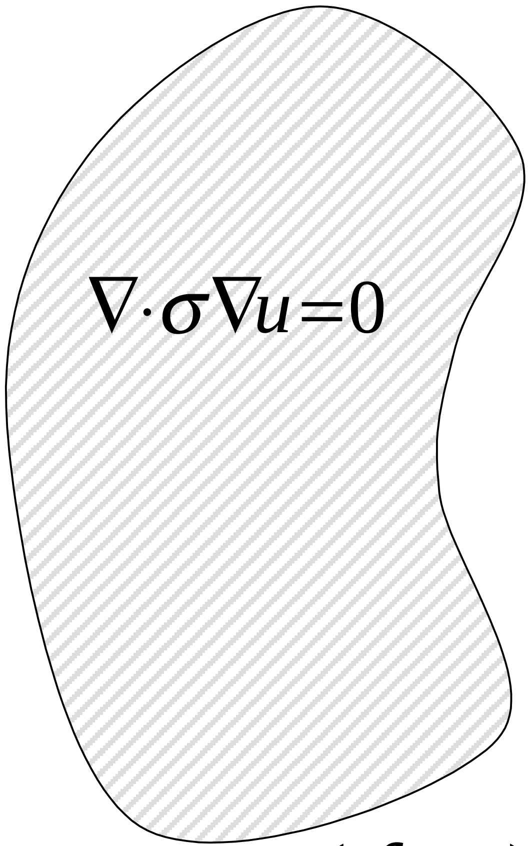
$$\textit{find } u(x) \textit{ in } \Omega \textit{ s.t.}$$

$$u = f \ \textit{ OR } \ \frac{\partial u}{\partial n} = g \ \textit{ on } \partial\Omega$$

Inverse Conductivity Problem

$$\textit{Find } \sigma(x) \textit{ from given}$$

$$u = f \ \textit{ AND } \ \frac{\partial u}{\partial n} = g \ \textit{ on } \partial\Omega$$

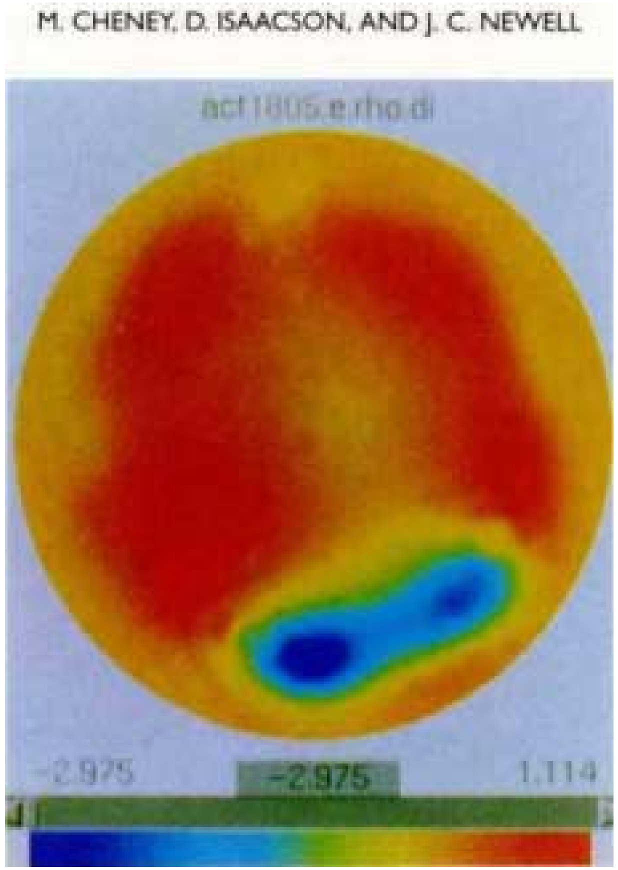


$$(f, g)\big|_{\partial\Omega}$$

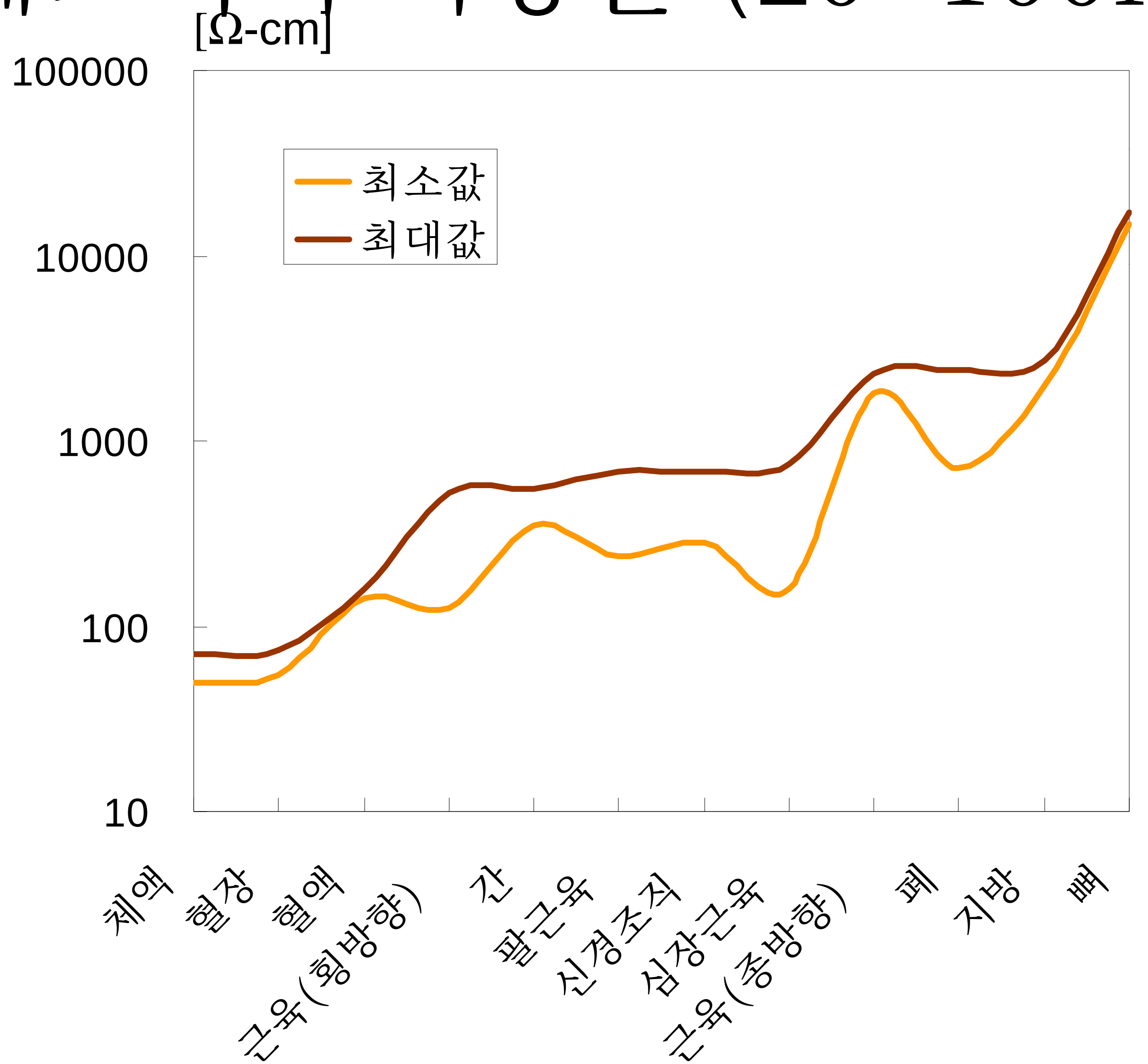
Electrical Impedance Tomography (EIT)

Impedance
~Voltage/Current

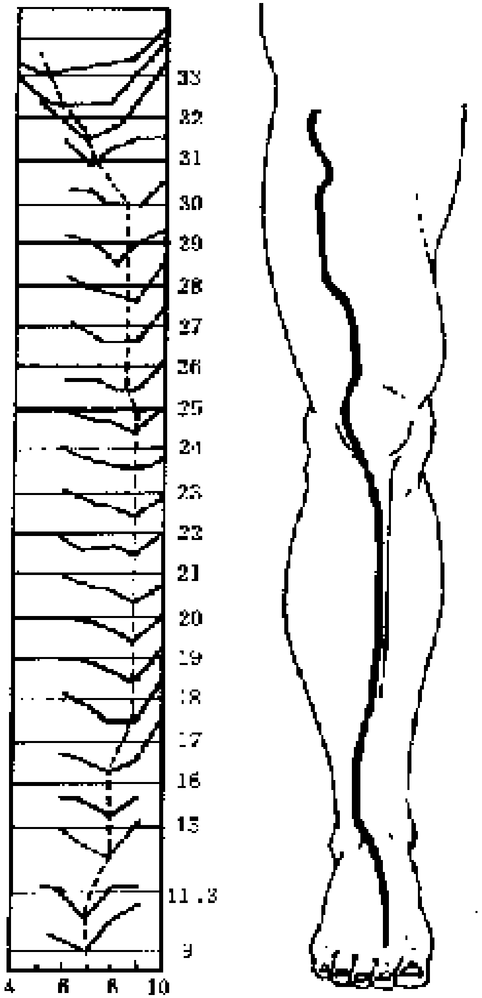
Tomography



생체조직의 저항률 (20 -100KHz)



경혈 및 경락의 전기적 특성

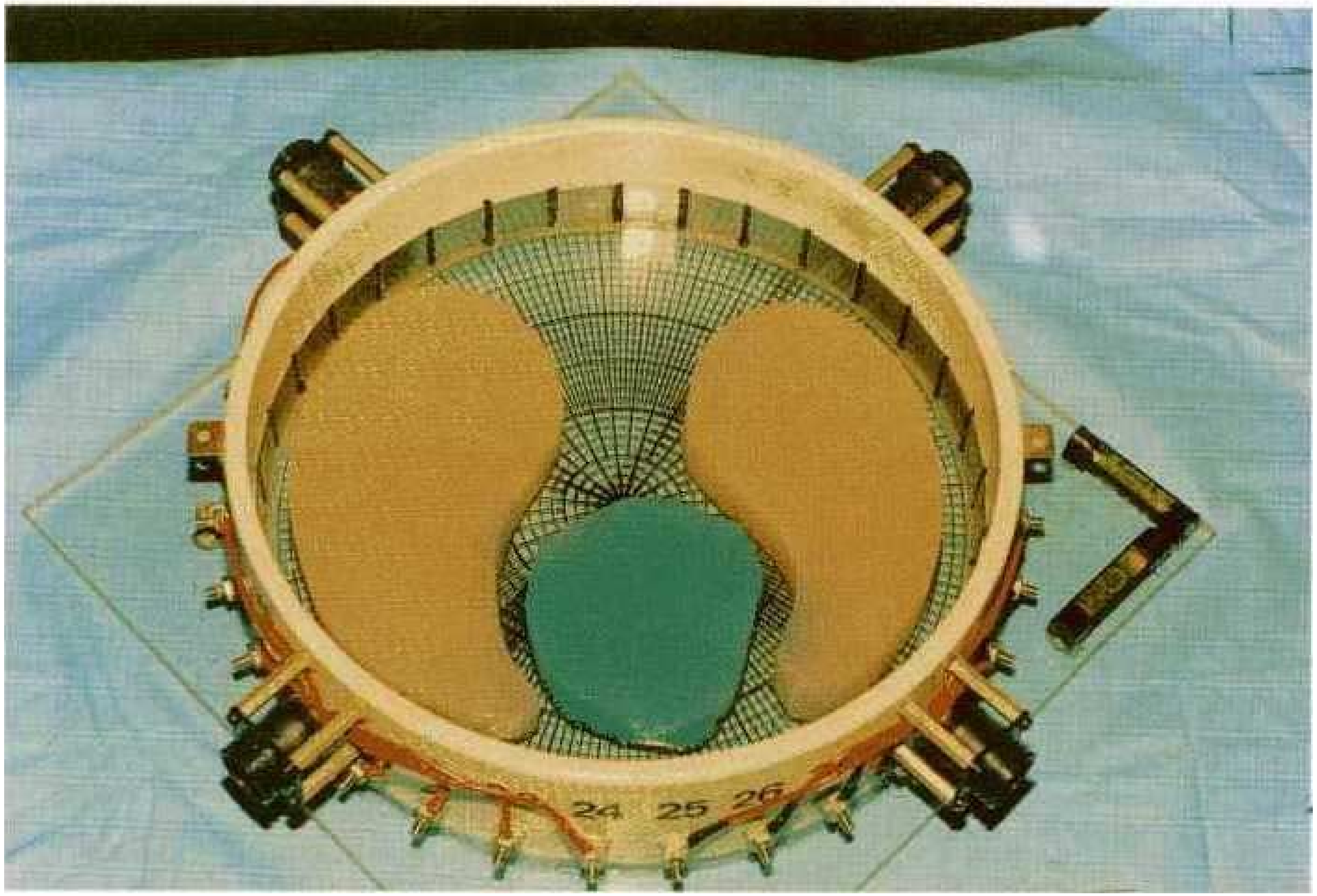


가 설

경혈과 경락 전기저항이
낮은 지점이다.

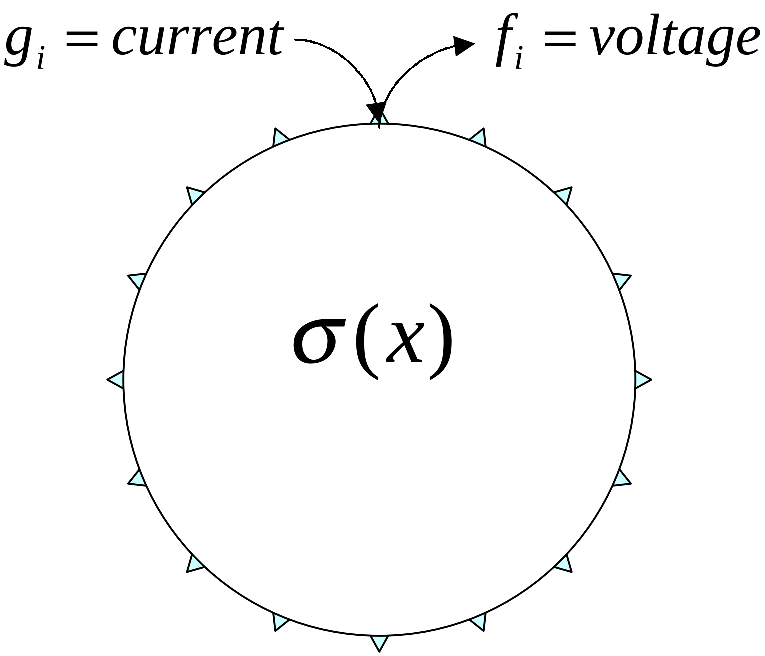
즉, 조직의
저항률이 낮다.

A model problem with heart and lung



Methods of EIT/ICP

Objective $\underset{\sigma}{Min}\sum\left|f_k - u_\sigma(x_k)\right|^2$ where $\nabla\cdot\sigma\ \nabla u_\sigma = 0,\ \dfrac{\partial u}{\partial n} = g$



- Single Measurement
  $\Rightarrow$ minimum information
- Many Measurement
  $\Rightarrow$ full information(?) on σ
- Infinite Measurement
  $\Rightarrow$ mathematical theory

Numerical Obstacles for EIT/ICP

Objective $\underset{\sigma}{Min} \sum \left| f(x) - u_\sigma(x) \right|_{L_2(\Omega)}$ where $\nabla \cdot \sigma \nabla u_\sigma = 0, \ \frac{\partial u}{\partial n} = g$

Guss $\sigma(x) \rightarrow$ $\boxed{\text{Solve } \nabla \cdot \sigma \nabla u_\sigma = 0, \ \frac{\partial u_\sigma}{\partial n} = g}$ $\rightarrow$ Find better $\tilde{\sigma}$

- Strongly nonlinear: $\sigma \in L_2(\Omega) \rightarrow u_\sigma \in L_2(\partial\Omega)$

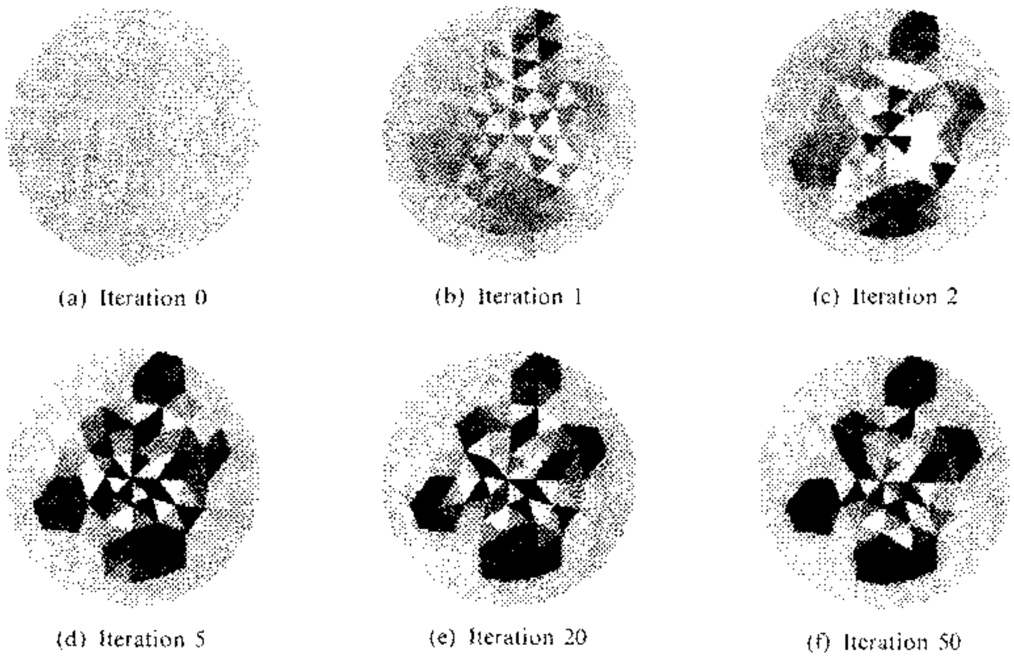
Solve $\nabla \cdot \sigma \nabla u_\sigma = 0, \ \frac{\partial u_\sigma}{\partial n} = g \rightarrow$ $\boxed{\text{Find better } \tilde{\sigma} \text{ using } u_\sigma - f}$

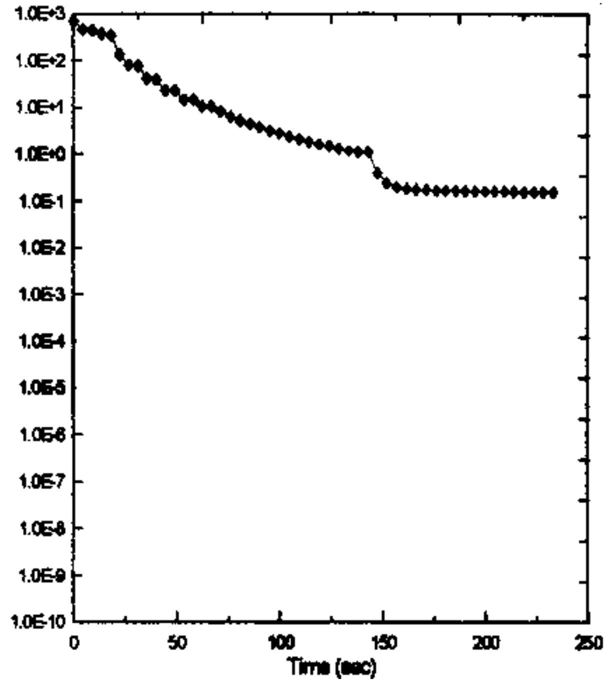
- Ill -posed problem: $\sigma \neq \tilde{\sigma} \rightarrow u_\sigma \approx u_{\tilde{\sigma}}$

Stop condition $u_\sigma - f \cong 0 \rightarrow$ $\boxed{\text{Regularity restrictiuon on } \sigma}$

Ill -posed Nonlinear

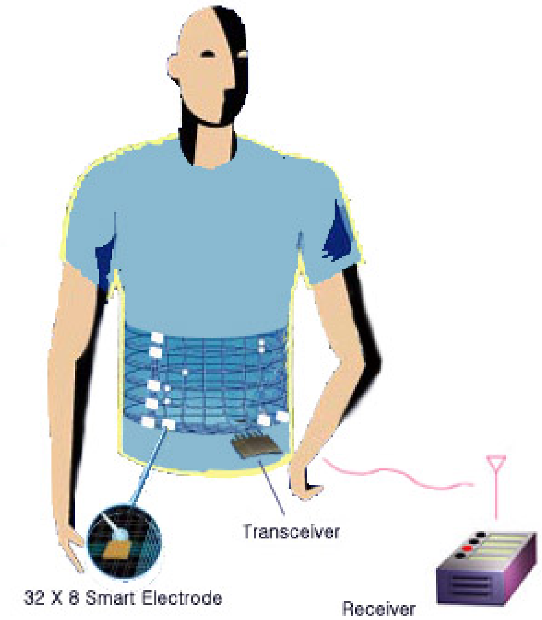
- 50 iterations

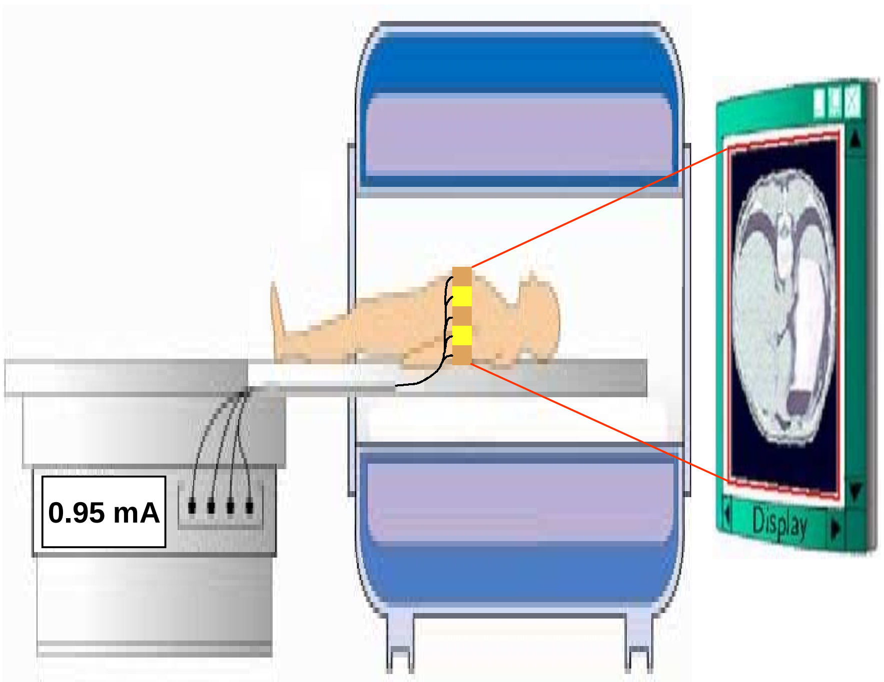


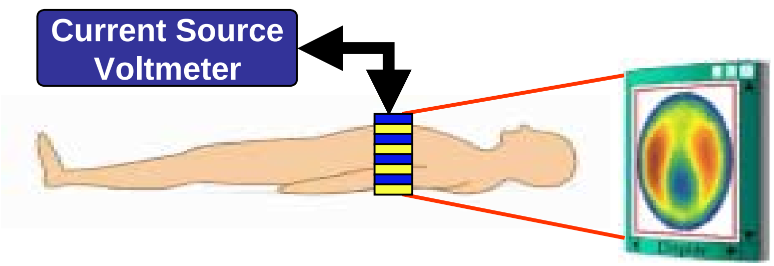


EIT and MREIT

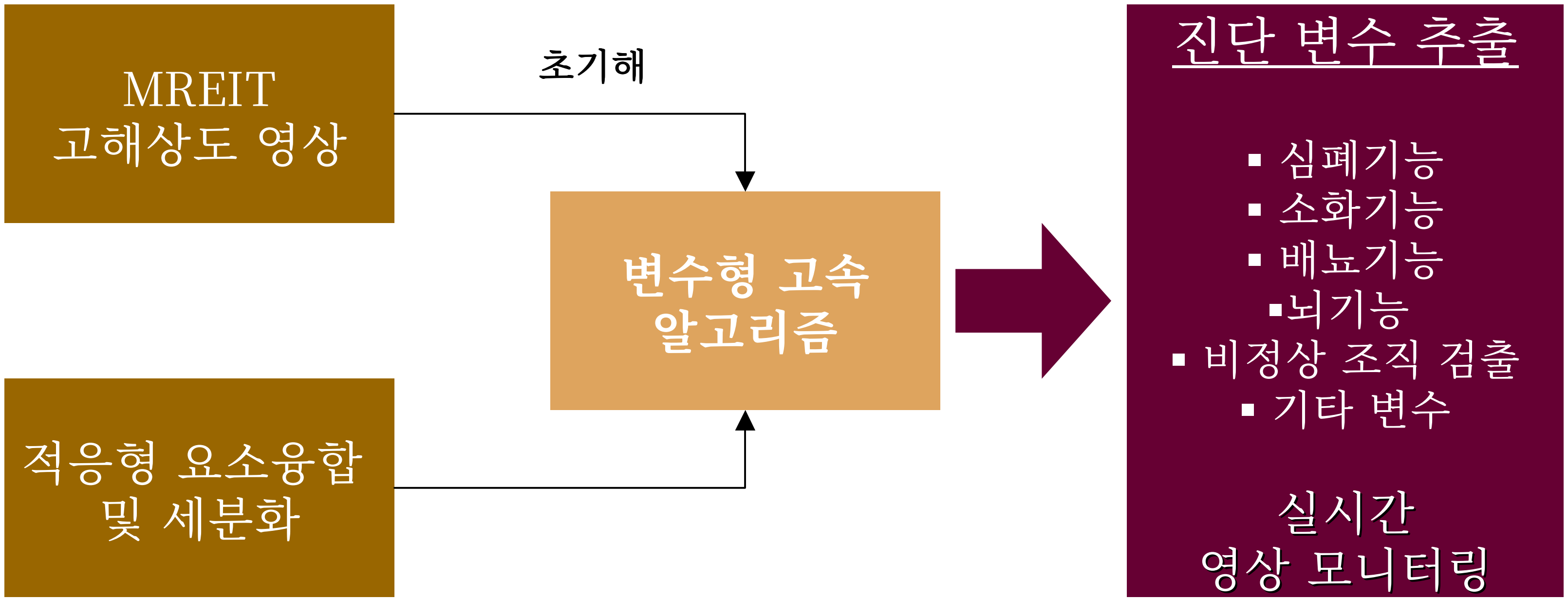
(Magenetic Resonace Electrical Impedance Tomography)





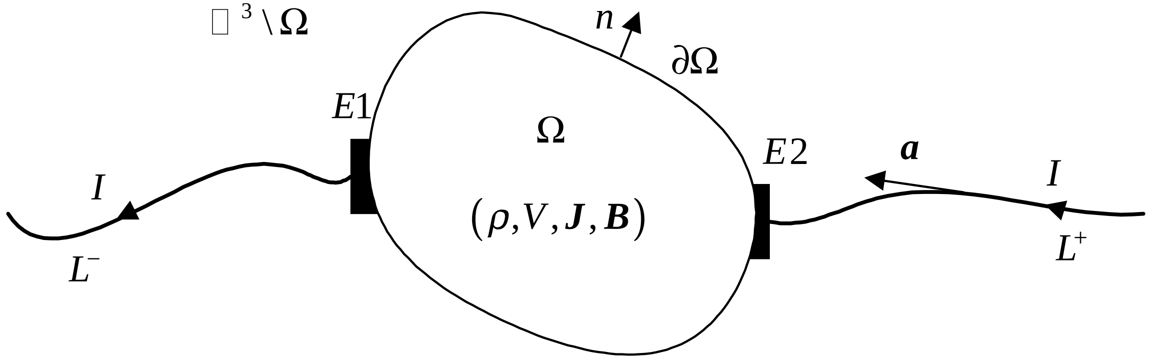


변수형 고속 영상복원 알고리즘



MRCDI 및 MREIT의 원리

(1) 문제정의



$$\nabla\left[\frac{1}{\rho(\mathbf{r})}\nabla V(\mathbf{r})\right]=0 \qquad \frac{1}{\rho}\frac{\partial V}{\partial n}=J \quad \text{on } \partial\Omega$$

$$\mathbf{J}(\mathbf{r})=-\frac{1}{\rho(\mathbf{r})}\nabla V(\mathbf{r})$$

$$\mathbf{B}_J(\mathbf{r})=\frac{\mu_0}{4\pi}\int_\Omega \mathbf{J}(\mathbf{r}')\times\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}dv'$$

$$\begin{aligned}\mathbf{B}_{I^\pm}(\mathbf{r}) &= \frac{\mu_0}{4\pi}\int_{\mathbb{R}^3\setminus\Omega} \mathbf{J}(\mathbf{r}')\mathbf{a}(\mathbf{r}')\times\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}dv' \\ &= \frac{\mu_0}{4\pi}\int_{L^\pm} I(\mathbf{r}')\mathbf{a}(\mathbf{r}')\times\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}dl'\end{aligned}$$

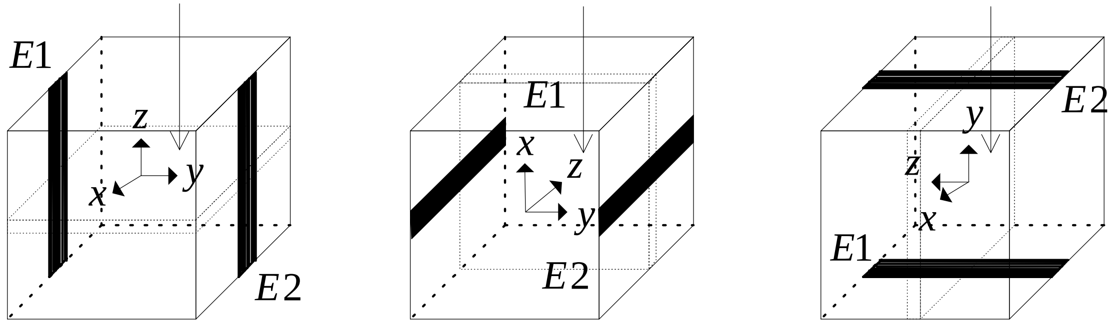
$$\mathbf{J}^B(\mathbf{r})=\frac{1}{\mu_0}\nabla\times\mathbf{B}(\mathbf{r})$$

(2) 자속밀도의 영상화

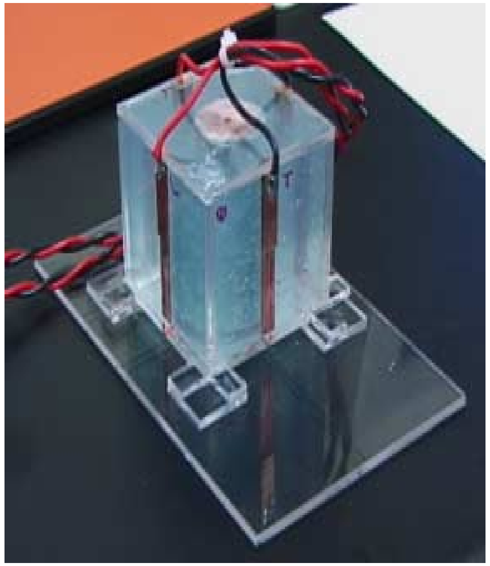
Transversal Imaging Slice

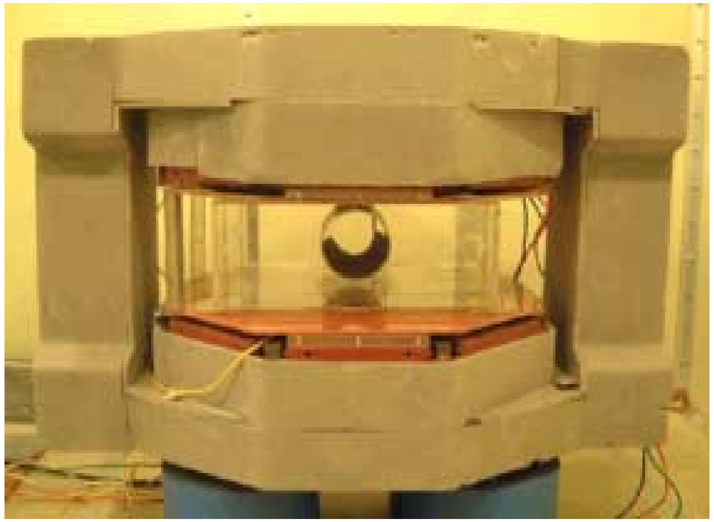
Three Sagittal Imaging Slices

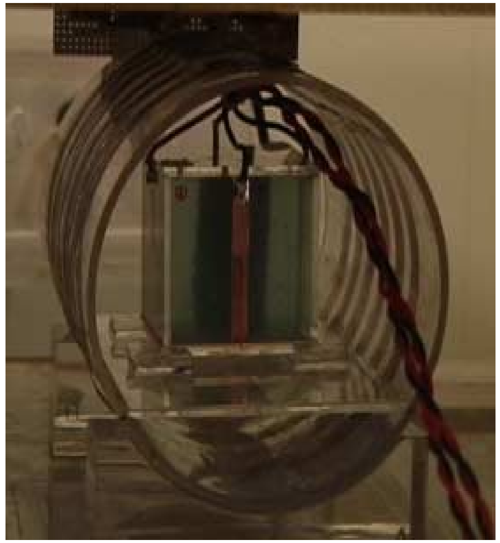
Three Coronal Imaging Slices

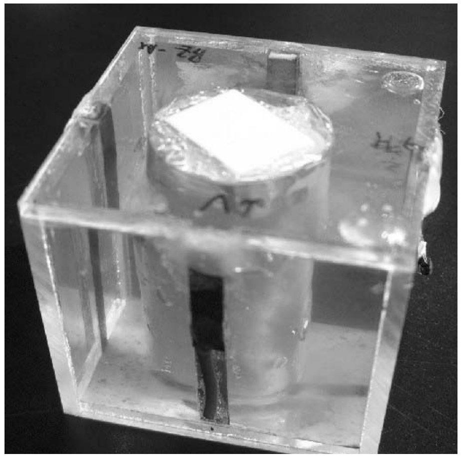


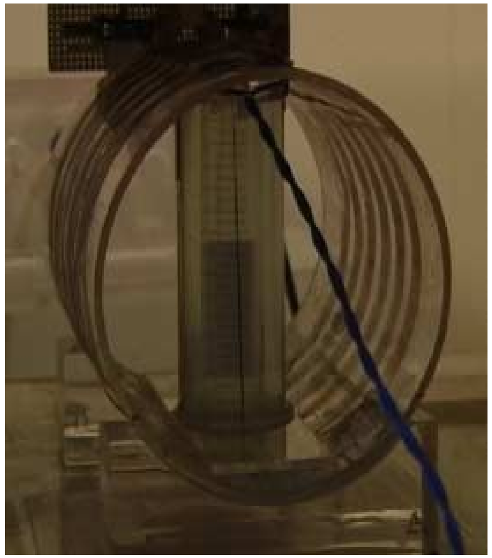
전류밀도 영상 실험용 팬텀





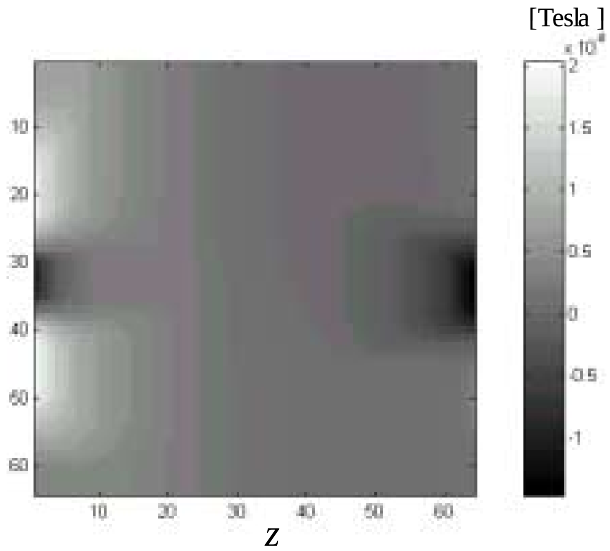




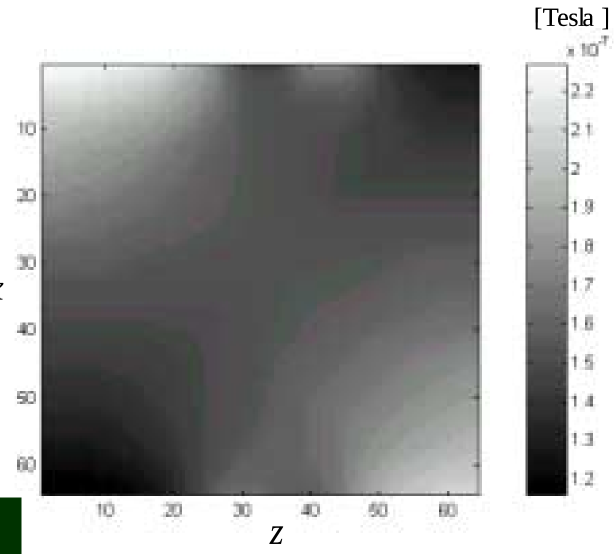


자속밀도 및 전류밀도 영상

B_x

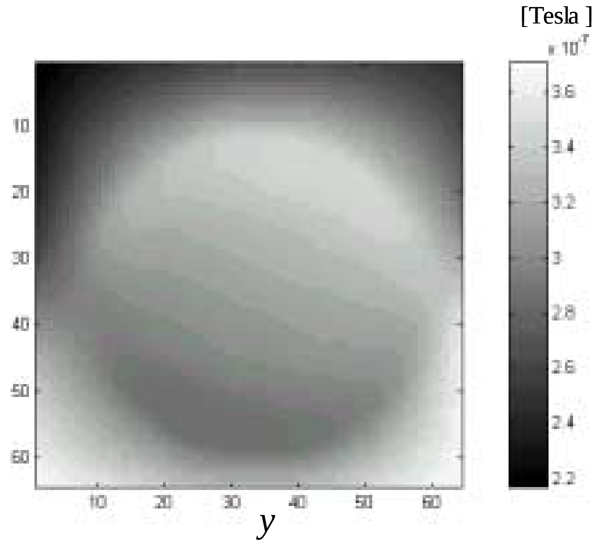


B_y

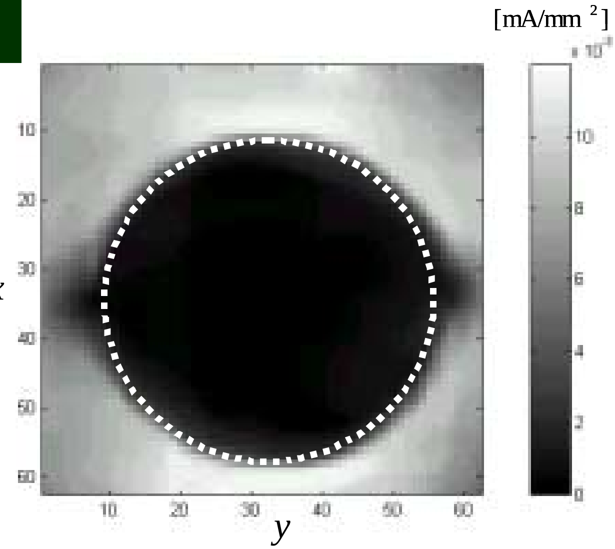


$$J = \frac{1}{\mu_0} \nabla \times B$$

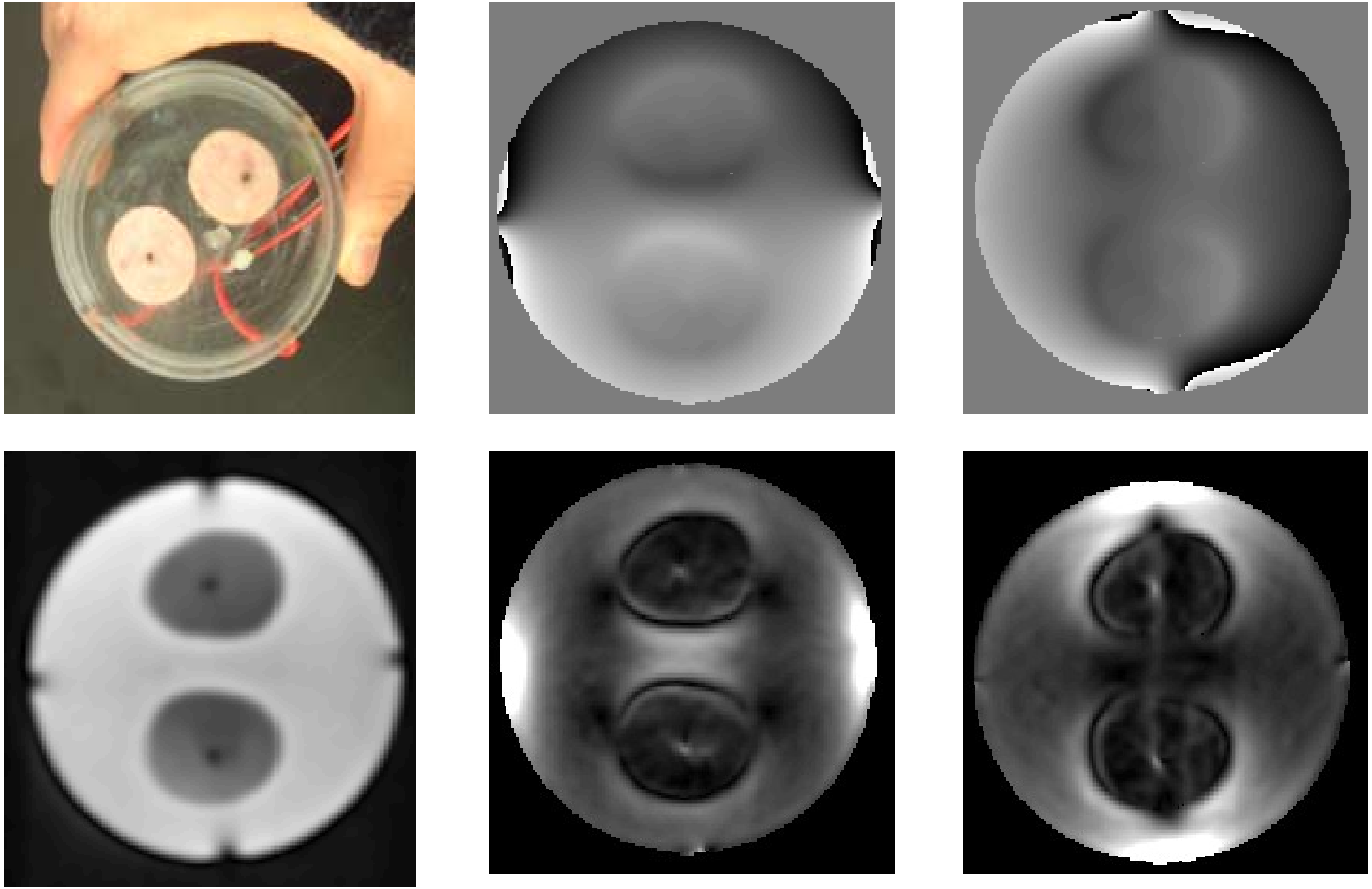
B_z



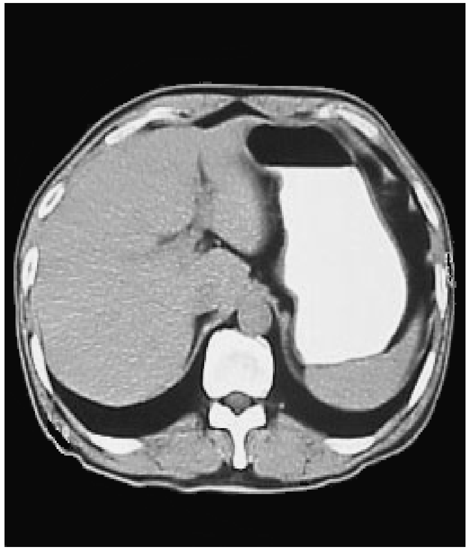
|J|



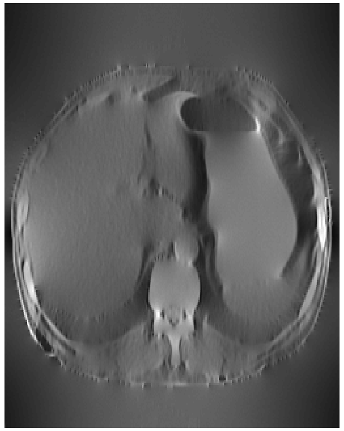
자속밀도 및 전류밀도 영상

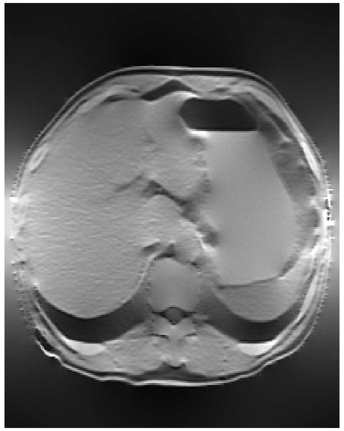


J-Substitution Algorithm

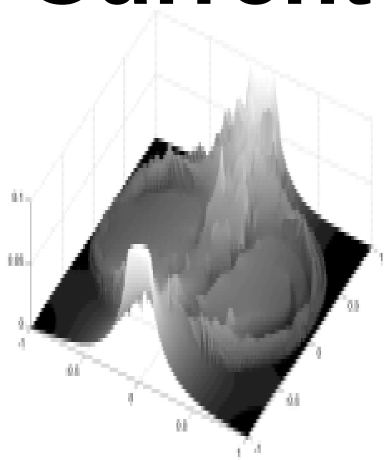


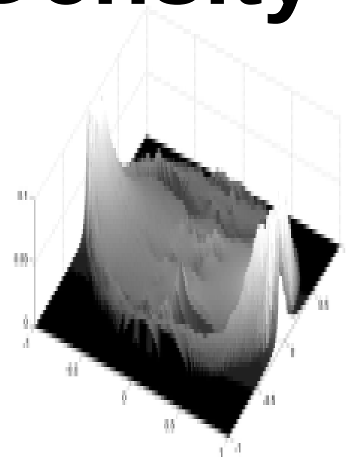
Original Conductivity

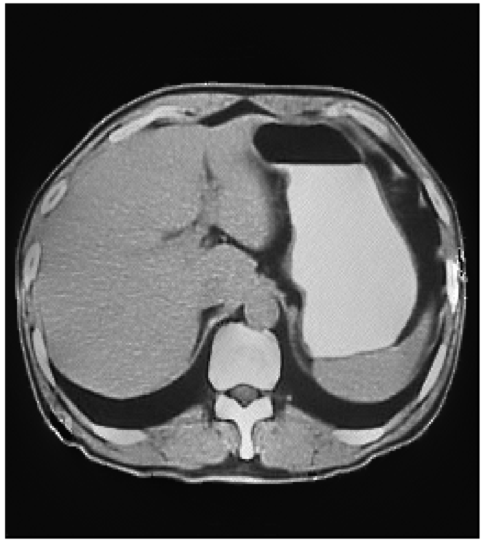




Current Density

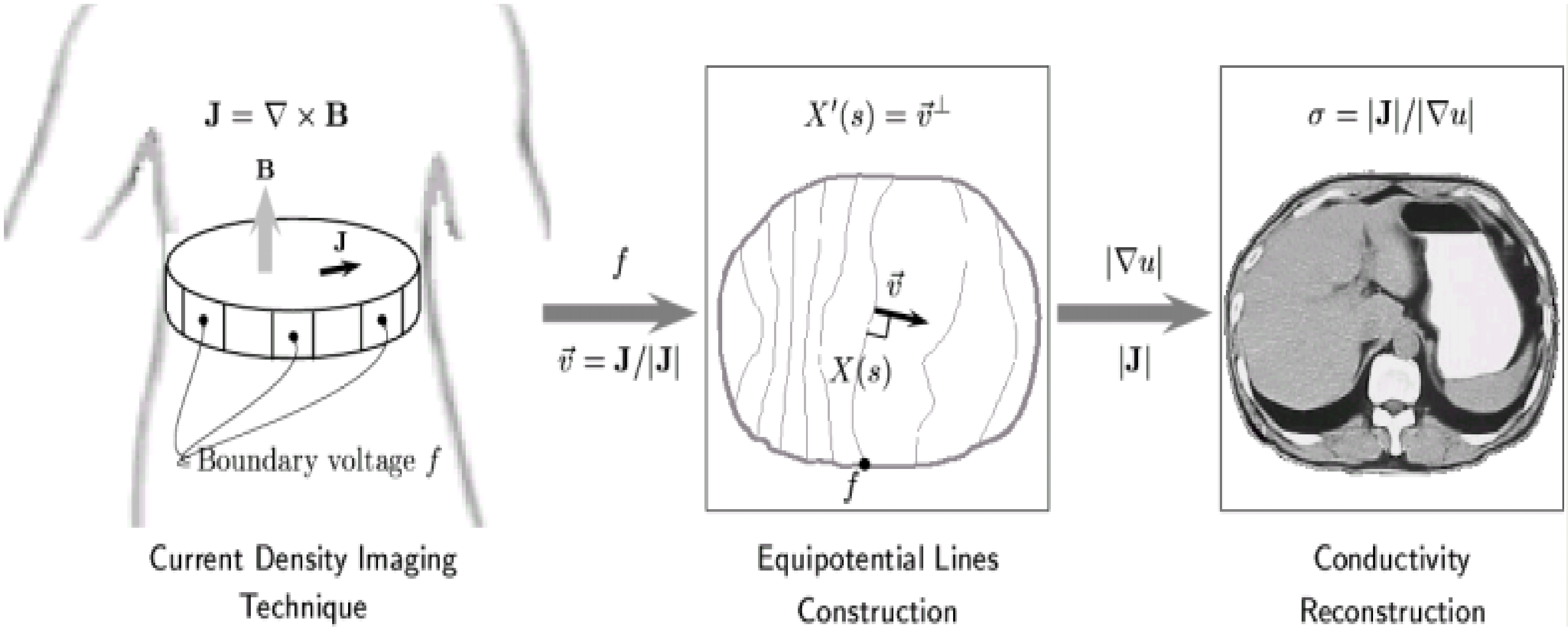






Reconstructed Conductivity

Schematic Diagram for Equi -potential Line Method



Numerical Algorithm

(a) Restoration of potential $u(x_t)$ in $x_t \in \Omega$: Find a boundary point $x_b \in \partial\Omega$ such that $u(x_t) = f(x_b)$ by solving a first order ordinary differential equation on equipotential line $X_t(s)$,

$$\frac{dX_t}{ds}(s) = \left(\frac{\mathbf{J}(X_t(s))}{|\mathbf{J}(X_t(s))|} \right)^{\perp} \quad \text{with} \quad X_t(0) = x_t \text{ and } X_t(s_f) \in \partial\Omega \tag{9}$$

where $(\cdot)^{\perp}$ denotes the counterclockwise right angle rotation of a vector.

(b) Reconstruction of conductivity $\sigma(x_t)$ in $x_t \in \Omega$: Calculate $|\nabla u(x_t)|$ using the reconstructed potential values of nearby points, then the $\sigma(x_t)$ is the ratio of $|\mathbf{J}|$ and $|\nabla u|$,

$$\sigma(x_t) = \frac{|\mathbf{J}(x_t)|}{|\nabla u(x_t)|}. \tag{10}$$

Equipotential Line Method

Step 0. **Given data**: Suppose $J_{ij} = \mathbf{J}(x_i, y_j)$ at all interior grid points $(x_i, y_j) \in \Omega$ and boundary potential data $f_k = f(x_k, y_k)$ at arbitrary boundary points $(x_k, y_k) \in \partial\Omega$ are given.

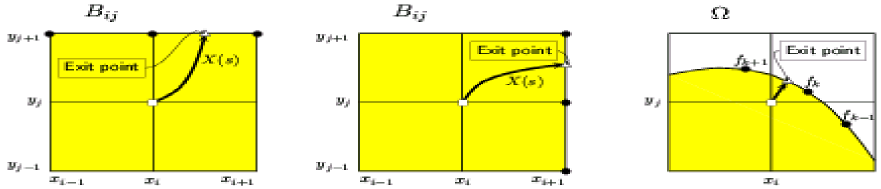
Step 1. **Restoration of potential** u_{ij}: Use a second order Runge-Kutta method with fixed step size, say $h = \frac{1}{4}\Delta x$ to solve the ordinary differential equation

$$X'_{ij}(s) = \vec{v}^{\perp}\left(X_{ij}(s)\right), \quad X_{ij}(0) = (x_i, y_j)$$

until it exits the bounding box $B_{ij} = [x_{i-1}, x_{i+1}] \times [y_{i-1}, y_{i+1}]$ or the domain Ω. Here $\vec{v}(x, y)$ is piecewise bilinear interpolation of v_{mn},

$$\begin{aligned}\Delta x_m \Delta y_n \vec{v}(x,y) = {} & (x_{m+1} - x)\left[(y_{n+1} - y)\ v_{m,n} + (y - y_n)\ v_{m,n+1}\right] \\ & + (x - x_m)\left[(y_{n+1} - y)\ v_{m+1,n} + (y - y_n)\ v_{m+1,n+1}\right]\end{aligned}$$

where $v_{mn} = \frac{J_{mn}}{|J_{mn}|}$ and $(x, y) \in [x_m, x_{m+1}{=}x_m{+}\Delta x_m] \times [y_n, y_{n+1}{=}y_n{+}\Delta y_n]$. And set target potential u_{ij} at the point marked with square to be second order interpolation of potentials at three neighboring points marked with heavy dots depending on the exit point in the following three cases.



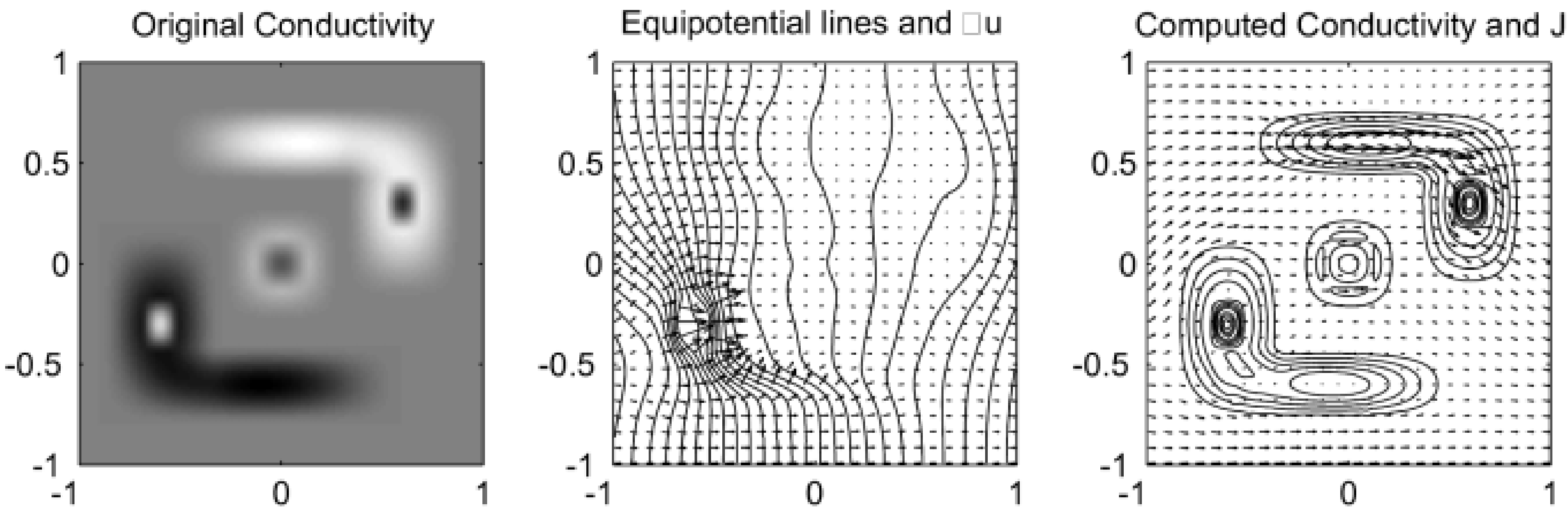
Step 2. **Reconstruction of conductivity** σ_{ij}: Once potential values at all interior grid points are known, ∇u can be approximated by

$$\nabla u_{ij} = \left(\frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x_i}, \ \frac{u_{i,j+1} - u_{i,j-1}}{2\Delta y_i}\right).$$

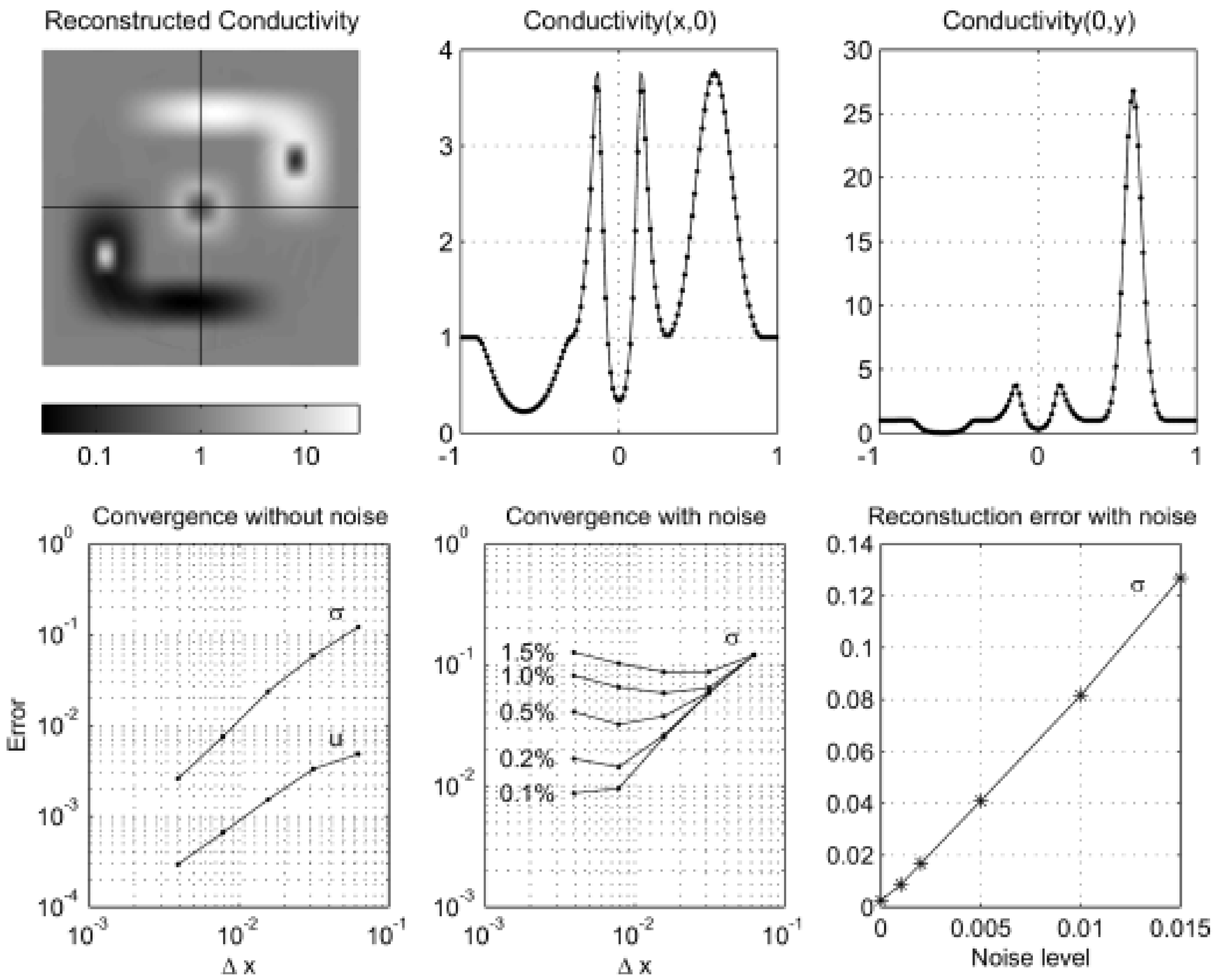
And conductivity at (x_i, y_j) is just the ratio of $|J_{ij}|$ and $|\nabla u_{ij}|$,

$$\sigma_{ij} = \frac{|J_{ij}|}{|\nabla u_{ij}|}.$$

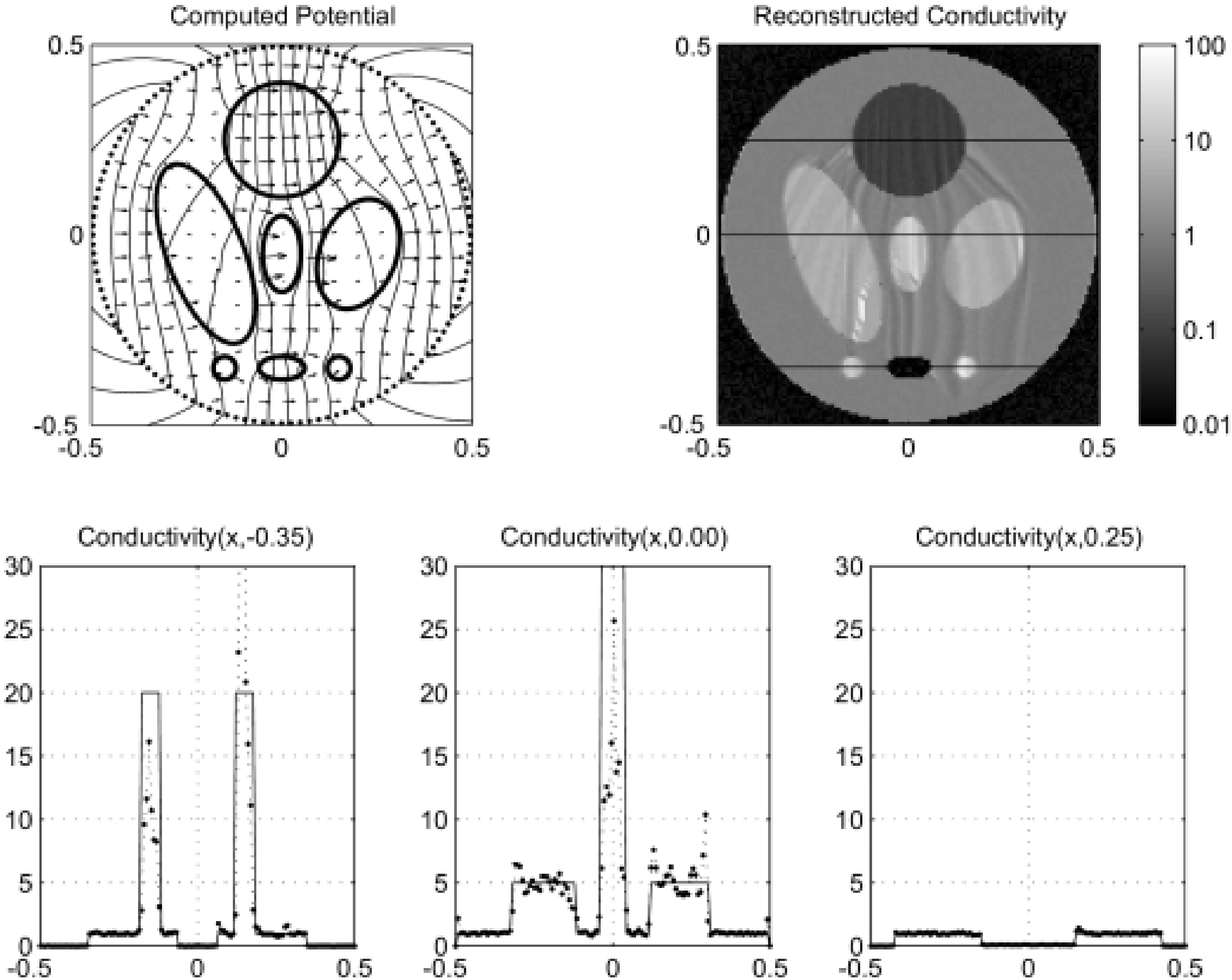
Smooth conductivity distribution



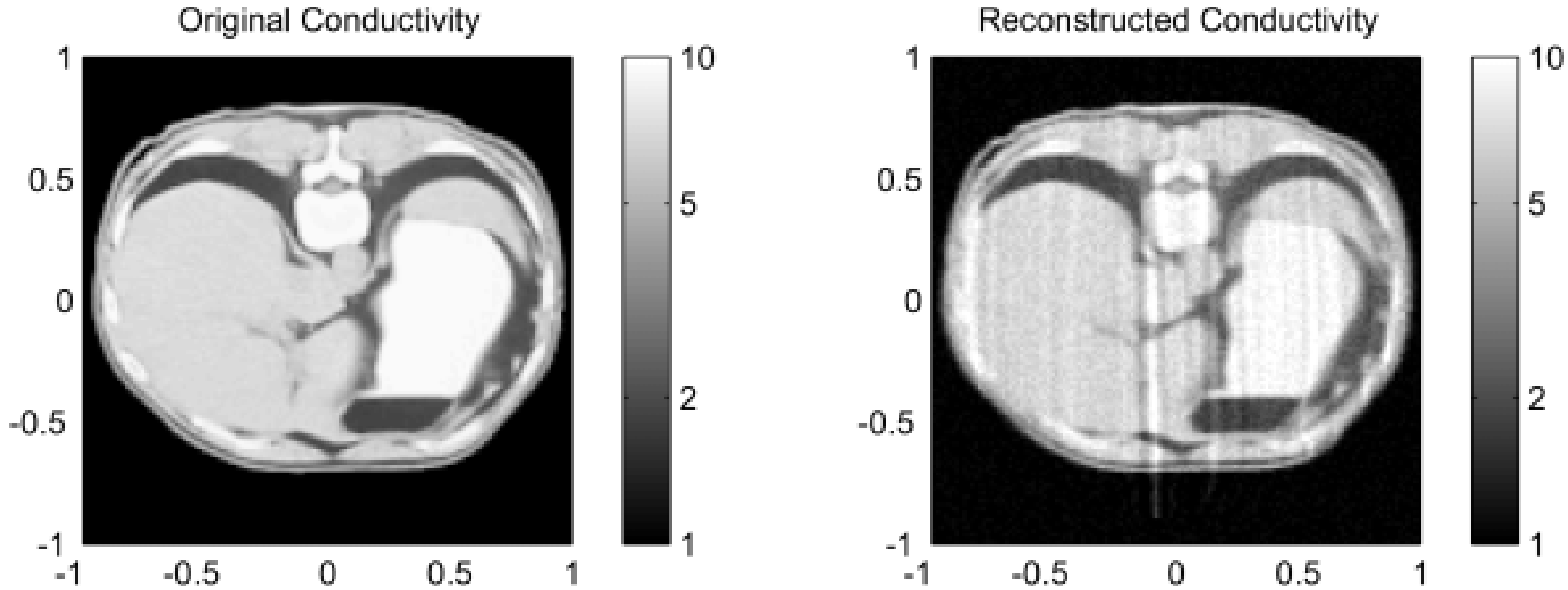
Conductivity Result with noisy J



Phantom with 1%Add+10%Mul Noise



Artificially generated Human Body
1%Add+10%Mul Noise



Conclusions and further works

- Fast, stable and efficient
- Well -posed (error~noise)
- Better interpolation method
  to handle discontinuous cases
- Noise handling for high conductivity
  ratio case where $|J| \sim 0$
- 3D extension is trivial
  But 3D data for J is expensive
  Usage of Bz instead of J is better