4.3.3 Sharpening Filters

- Objectives :
  - to highlight fine detail
  - to enhance detail that has been blurred

- various methods according to applications :

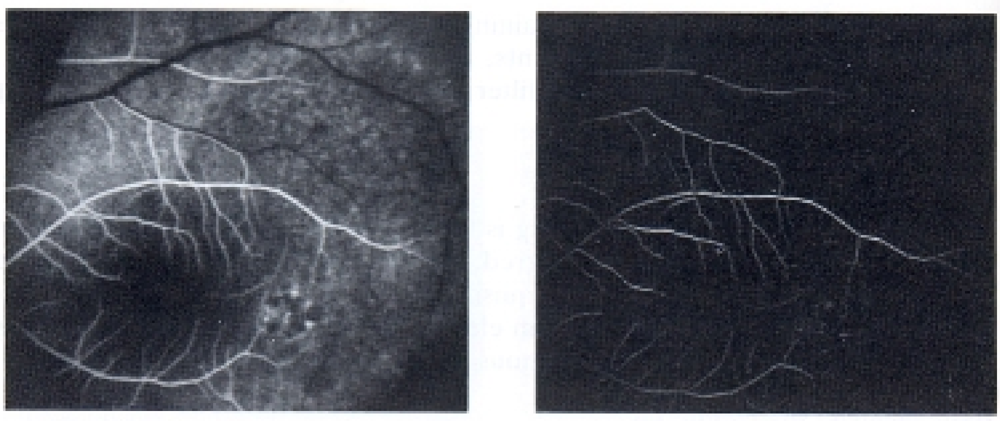
1) Basic highpass spatial filtering

- Impulse response :
  - positive coeffs. near its center
    negative coeffs. in the outer periphery

3×3 sharpening filter

$$\frac{1}{9} \times \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

- HPF :
  - eliminate the zero frequency term
  - → reduce the average gray-level value to zero
  - → reduce significantly the global contrast of image



(a) (b)

✓ dark background, enhance edge

2) High-boost filtering :

- High pass = original – lowpass

- High boost (high freq. emphasis) :

  - high boost = A(original) – Lowpass
    = (A-1)(original) + original-Lowpass
    = (A-1)(original) +Highpass

    where A : amplification factor

  - if A=1 → standard highpass result

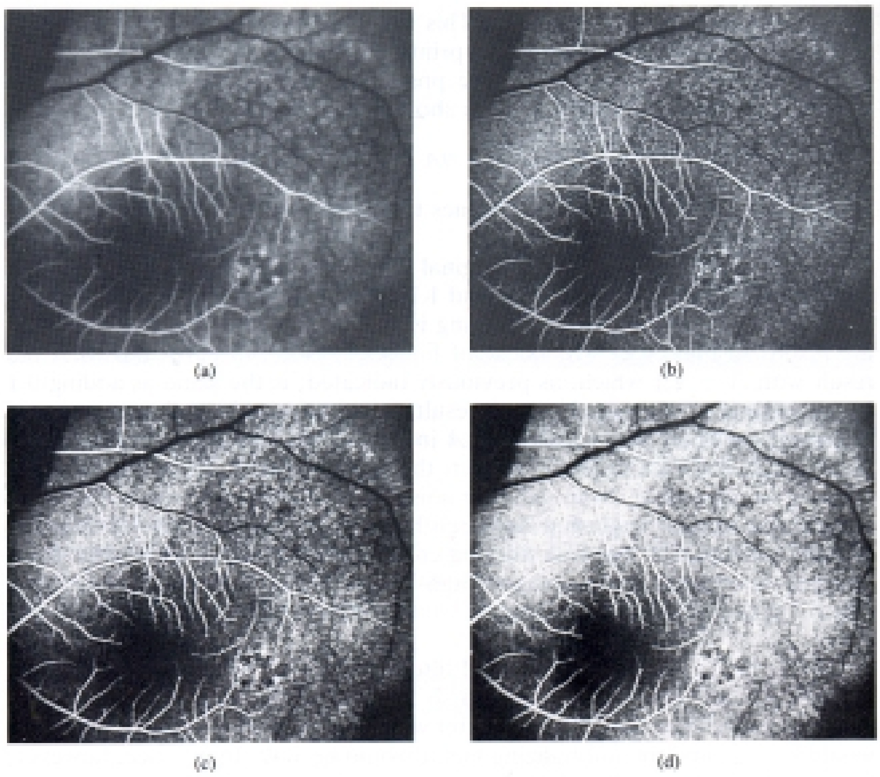
  - when A>1
    - ✓ part of original is added back to the highpass result
    - ✓ restore partially the low freq. components lost by HPF
    - ✓ the result : the high-boost image looking more like the original image
    - ✓ relative degree of edge enhancement : dependent on the value of A

  - implementation ;

    - ✓ the center weight of mask
      w=9A-1

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline -1 & -1 & -1 \\ \hline -1 & w & -1 \\ \hline -1 & -1 & -1 \\ \hline \end{array}$$

ex.)



A: increase → background : brighter

3) Derivative filters :

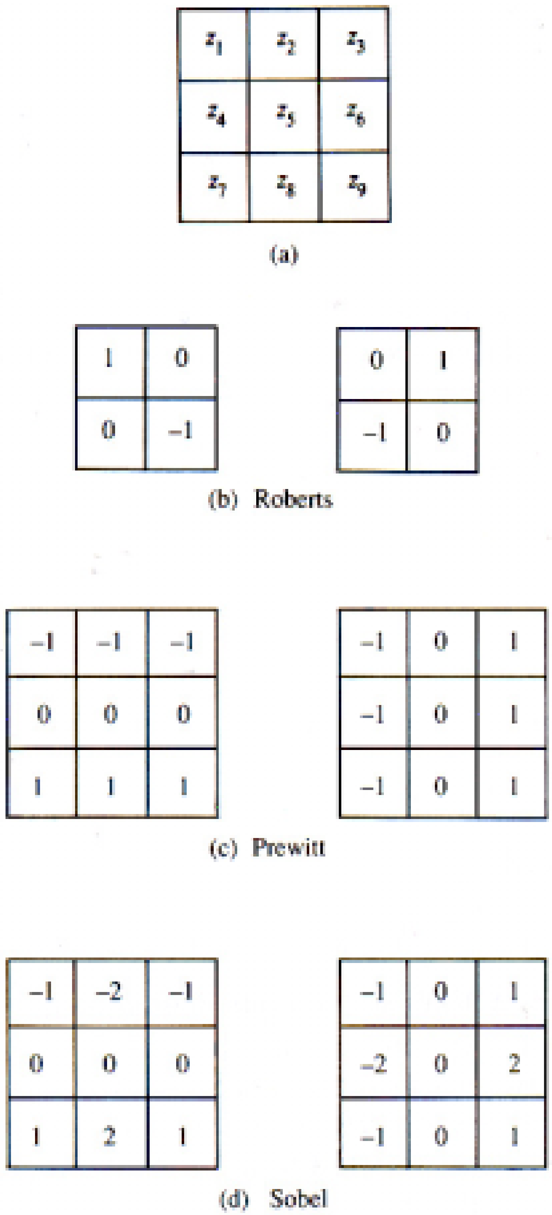
- Low pass :
  - Integration: analogous to average
  - tend to blur detail

- Differentiation :
  - effect opposite to integration
  - sharpen an image
  - gradient, for f(x,y)
  - $\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$

- magnitude ;

$$\nabla f = mag(\nabla \mathbf{f}) = \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2}$$

●



(a) (b) Roberts (c) Prewitt (d) Sobel

- approximately, at point z_5 ;

$$\frac{\partial f}{\partial x} = z_5 - z_8\,,\quad \frac{\partial f}{\partial y} = z_5 - z_6\,, \nabla f \approx [(z_5 - z_8)^2 + (z_5 - z_6)^2]^{1/2}$$

- another approach; Use of cross difference

  $\nabla f \approx [(z_5 - z_9)^2 + (z_6 - z_8)^2]^{1/2}$

  $\Rightarrow$ implemented using masks of size 2×2

  ex.) (b) Roberts cross-gradient operators

- 3×3 mask ;

  $\nabla f \approx |(z_7 + z_8 + z_9) - (z_1 + z_2 + z_3)| + |(z_3 + z_6 + z_9) - (z_1 + z_4 + z_7)|$

  (c) Prewitt operator

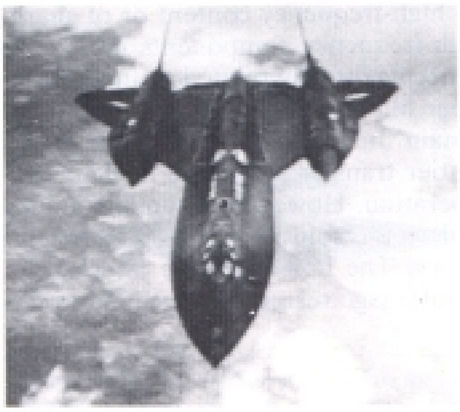
  (d) Sobel operator

- all masks ;

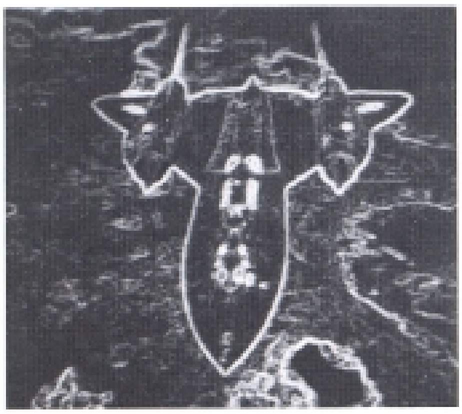
  : sum of all weights in mask : 0

  → mean of HPF (for even values of gray levels → zero output)

ex.) Edge enhancement by gradient technique



(a)



(b)

(c)

(d)

(a) original image
(b) image obtained by using Prewitt mask
(c) image obtained by setting to maximum white (255) any gradient value greater than 25 (that is, approximately 10% or higher of the highest possible gray-level value in the image)
Any point which do not meet this criterion : set equal to its original value
→ restoring the background white and enhancing prominent edge
(d) image obtained in the same way as (c), except that point for which the gradient did not exceed 25 : set to zero (black)

4.4 Enhancement in the Frequency Domain

- procedure :

$$f(x,y) \xrightarrow{F.T} F(u,v) \xrightarrow[H(u,v)]{filtering} G(u,v) = F(u,v)\cdot H(u,v) \xrightarrow{IFT} g(x,y)$$

- spatial :
  - simpler implementation
  - higher speed operation

  (both: Than freq. domain processing)

  → frequently used

- understanding of freq. domain concept :
  - essential to the solution of many applications.

4.4.1 Lowpass filtering

- high freq. components in image :
  - edge, sharp transitions (noise)
  - blurring : obtained by LPF

- $G(u,v) = H(u,v)\cdot F(u,v)$

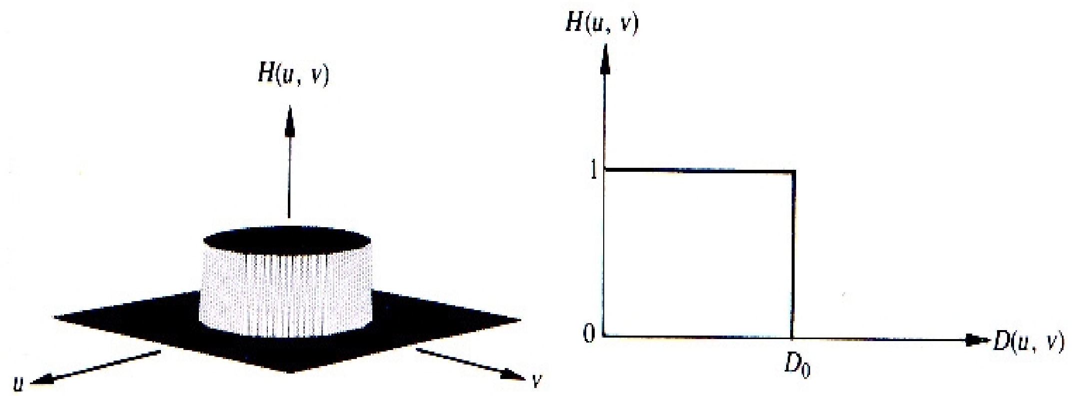
1)ideal filter

- 2-D ideal LPF :

$$H(u,v) = \begin{cases} 1 & if \quad D(u,v) \le D_0 \\ 0 & if \quad D(u,v) > D_0 \end{cases}$$

where D_0 : nonnegative value

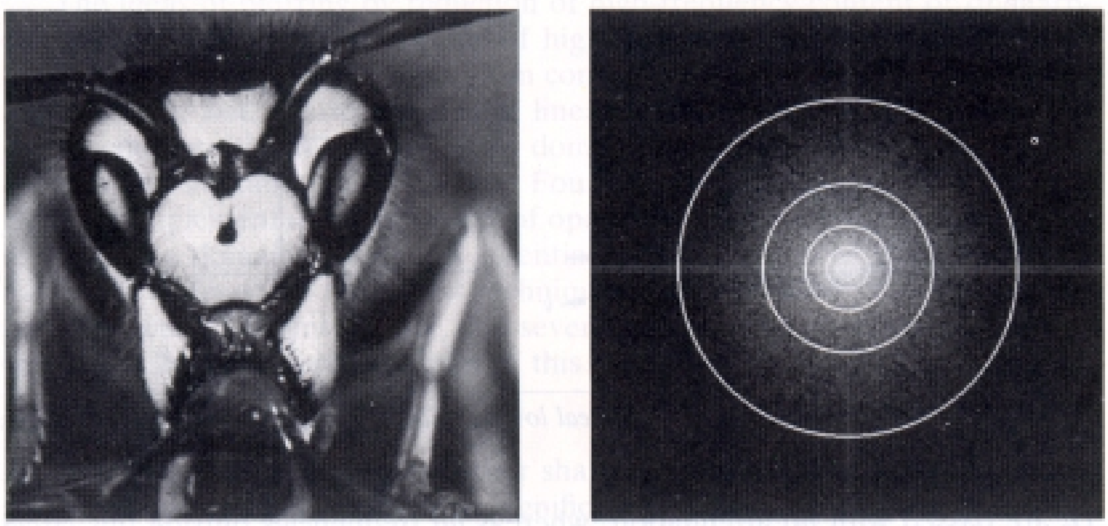
$D(u,v)$: distance from point (u,v) to origin that is $D(u,v) = (u^2 + v^2)^{1/2}$



(a) (b)

- radically symmetric about the origin
- D_0 : cutoff freq.

ex.)



(a) (b)

(a) image with a variety of detail from fine to coarse

- total power :
  - $P_T = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} P(u,v)$
  - β percent of power

    $\beta = 100 \left[\sum_u \sum_v P(u,v) / P_T \right]$

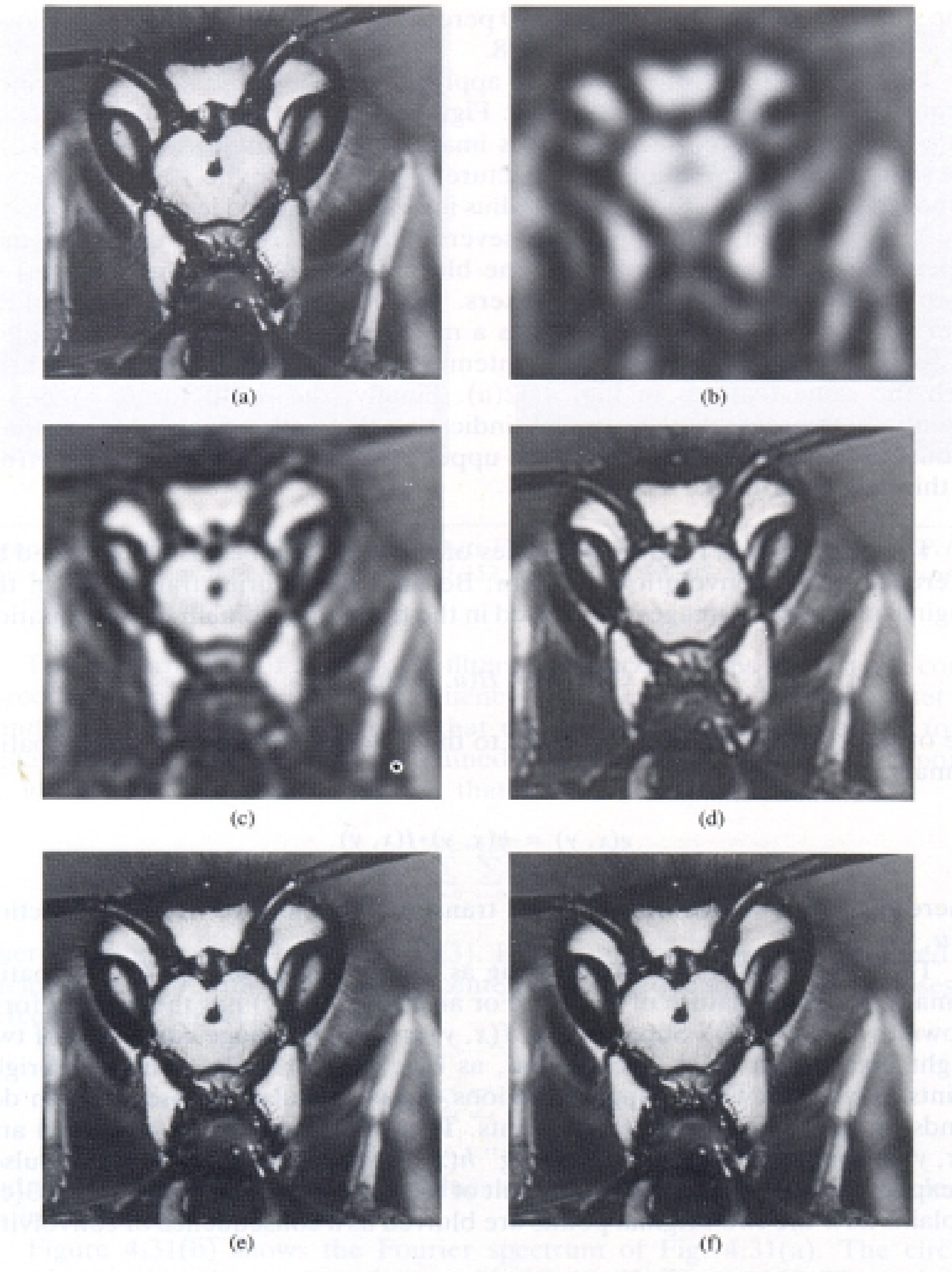
    Sum taken over value of (u,v) inside circle

- In (b) :

  radius of circles : 8, 18, 43, 78, 153

  β percent of power : β =90, 93, 95, 99, 99.5

- the results of applying ideal LPF with cutoff freqs. At the radii given above



(a) original image

(b) β =90, clear indication that most of the sharp detail information is contained in the 10% power removed by the filter

(c) β =95, Still severe ringing

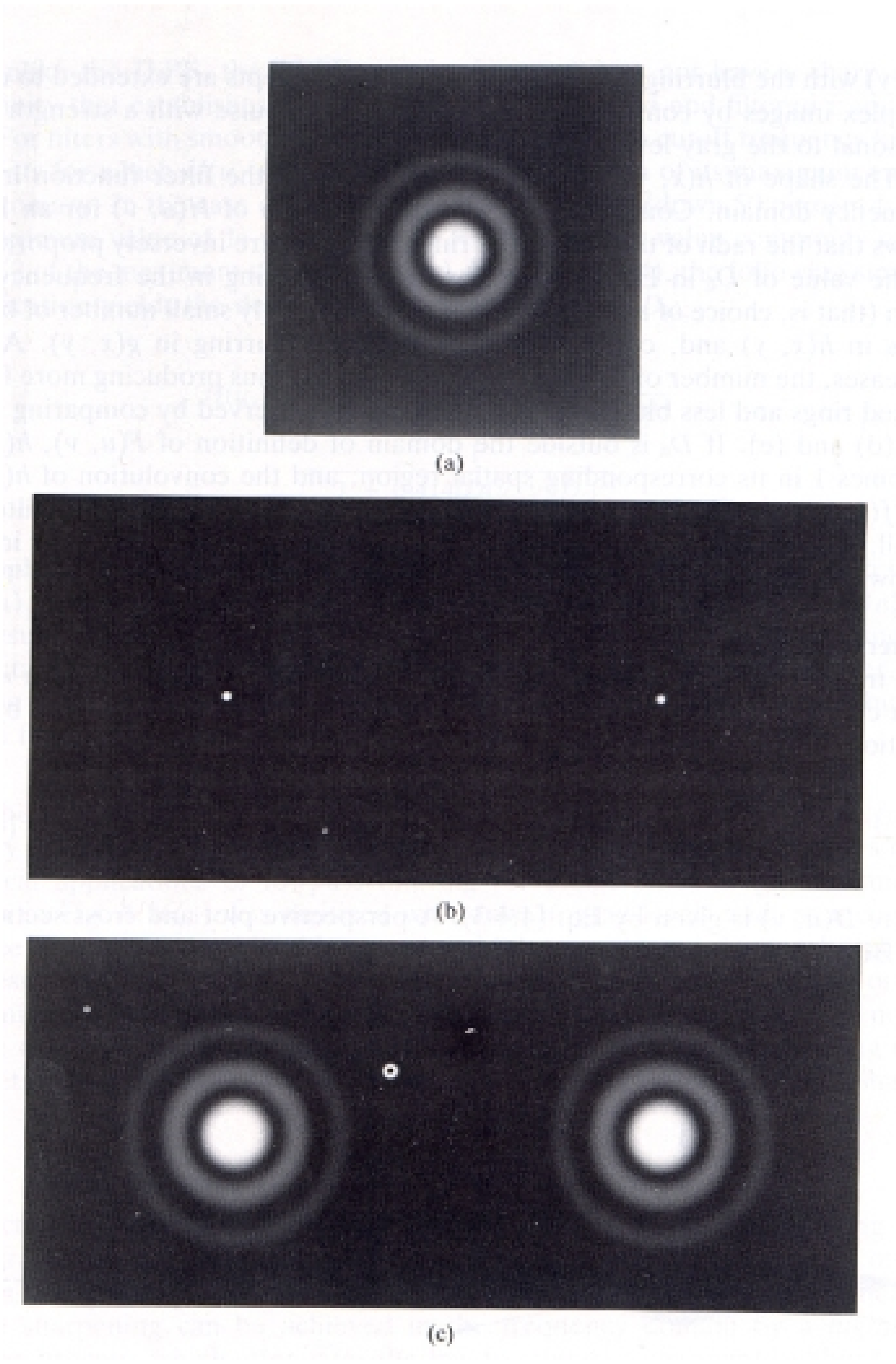
(d) β =99, Mild degree of blurring

(e) β =99.5, essentially the same as the original

- blurring and ringing properties of ILPF :

$$G(u,v) = H(u,v) \cdot F(u,v)$$

$$g(x,y) = h(x,y) * f(x,y)$$

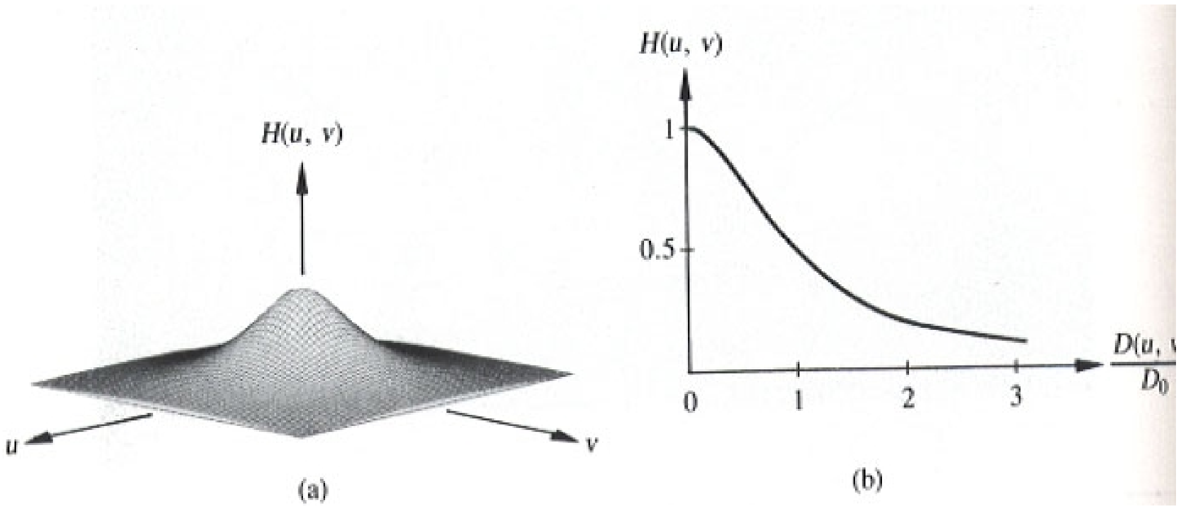


(a) blurring function, (b) two bright spots, (c) $g(x,y) = h(x,y) * f(x,y)$

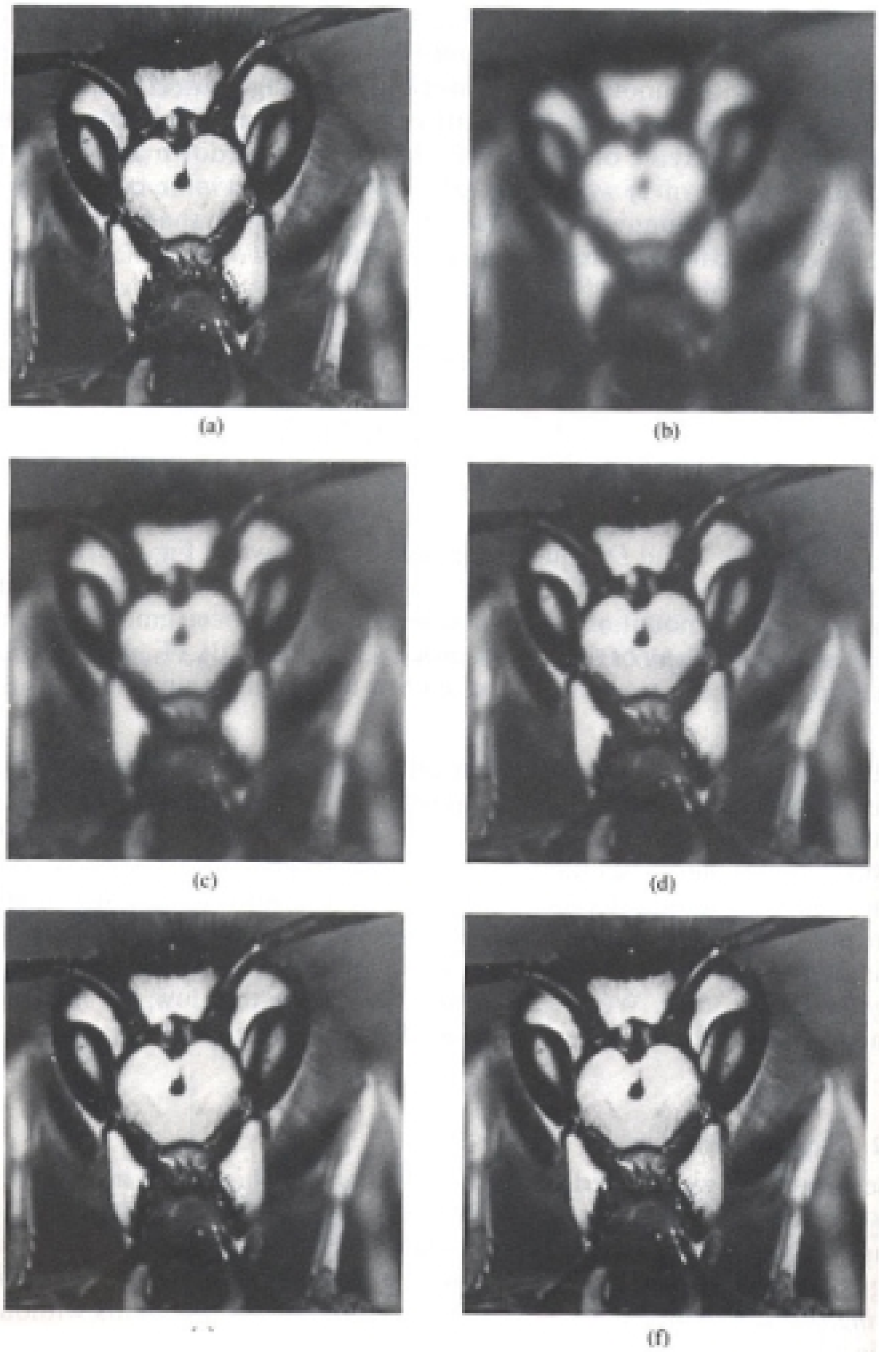
2) Butterworth filter

- to reduce ringing artifacts : no sharp discontinuity

$$H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}}$$



(a) butterworth filter (b) radial cross section

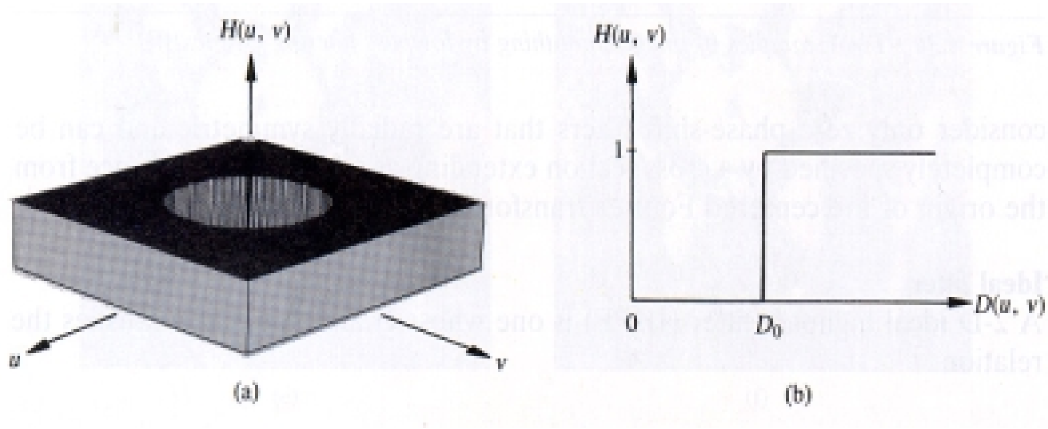


→ smooth transition in blurring

→ no ringing effect

4.4.2 high pass filter

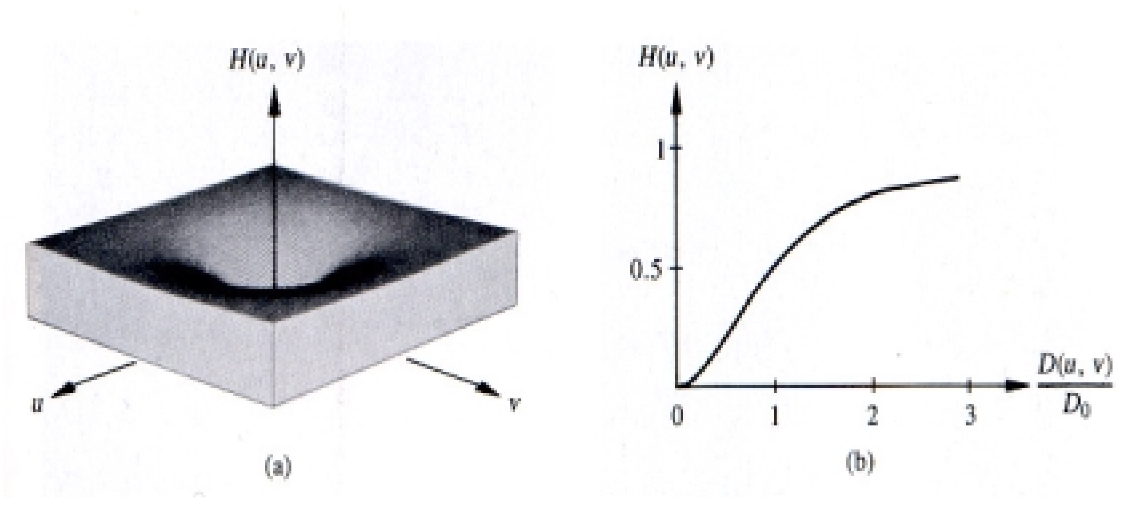
- IHPF ;



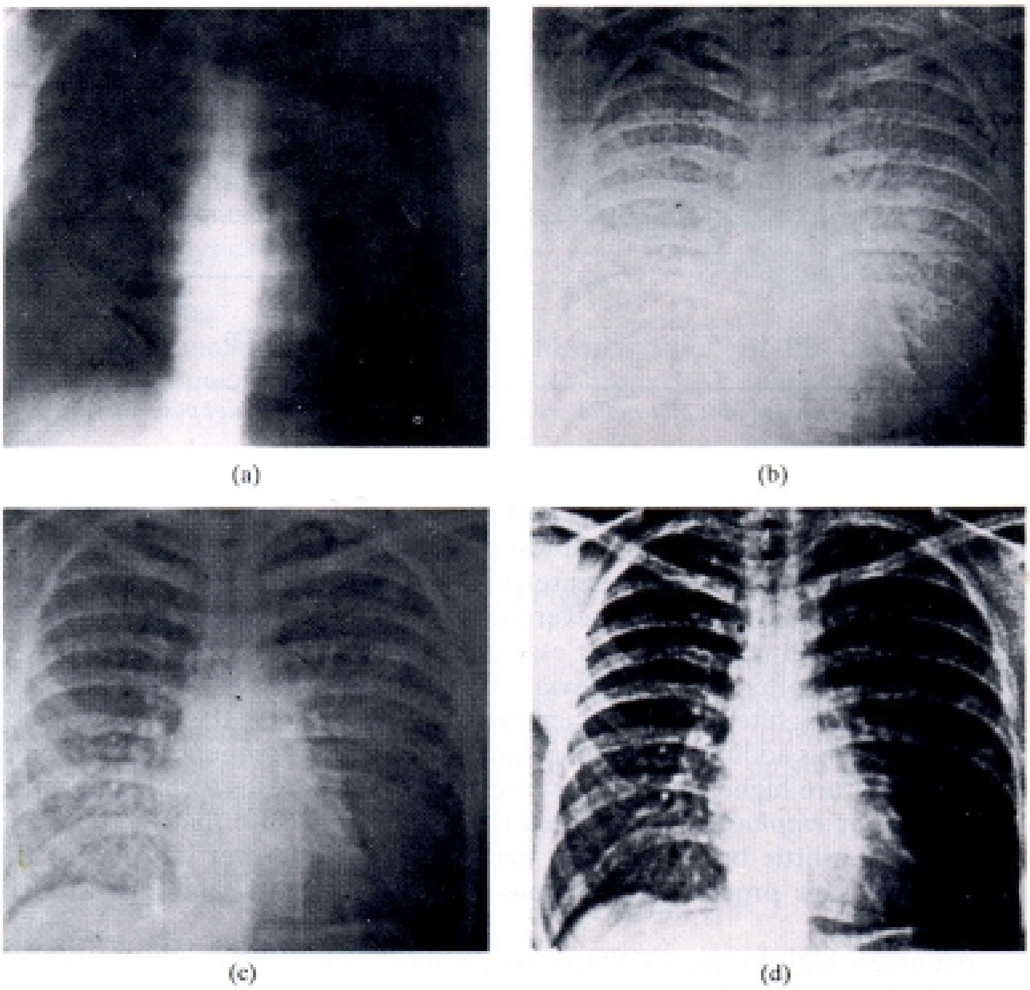
2) Butterworth filter

- BHPF :

$$H(u,v)=\frac{1}{1+[D_0/D(u,v)]^{2n}}$$



ex.) BHPF : order 1



(a) origin
(b) high pass filtered image
(c) high boost (high freq. emphasis)
(d) high boost + histogram equalization

3) Homomorphic Filter (Multiplication)

- Image :
  - $f(x,y) = i(x,y)r(x,y)$

illumination ($i(x,y)$), reflection ($r(x,y)$)

- Procedure :
  - taking logarithm:

$$z(x,y) = \ln f(x,y)$$

$$= \ln i(x,y) + \ln r(x,y)$$

  - FT

$$Z(u,v) = I(u,v) + R(u,v)$$

  - Filtering

$$S(u,v) = H(u,v)Z(u,v)$$

$$= H(u,v)I(u,v) + H(u,v)R(u,v)$$

  - IFT

$$s(x,y) = i'(x,y) + r'(x,y)$$

  - Take inverse operation of log -> exponent

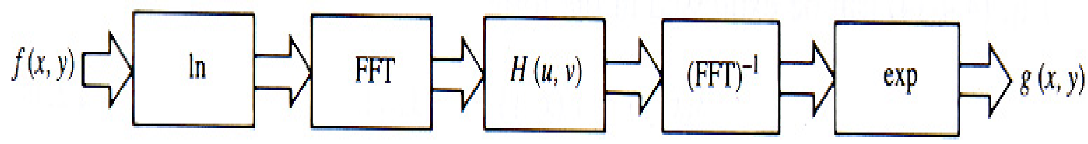
$$g(x,y) = \exp[s(x,y)]$$

$$= \exp[i'(x,y)] \cdot \exp[r'(x,y)]$$

$$= i_0(x,y) r_0(x,y)$$

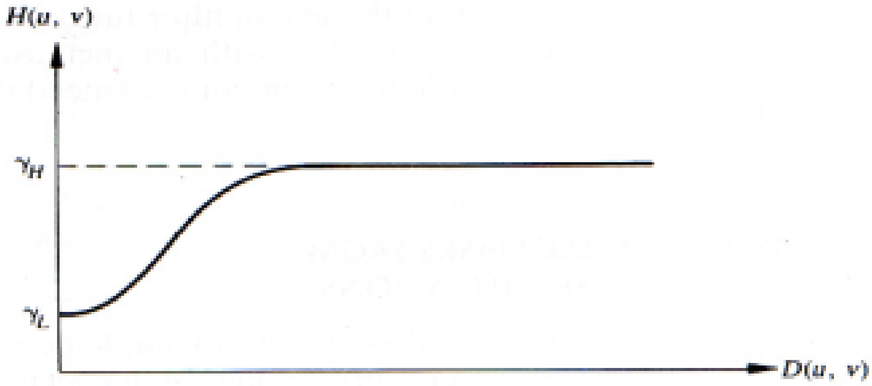
$i_0(x,y), r_0(x,y)$: illumination and reflection components of output image

- Homomorphic system :



  - H(u,v) : homomorphic filter

- Filter H(u,v) affects the low – and high–freq. components of FT in different ways :

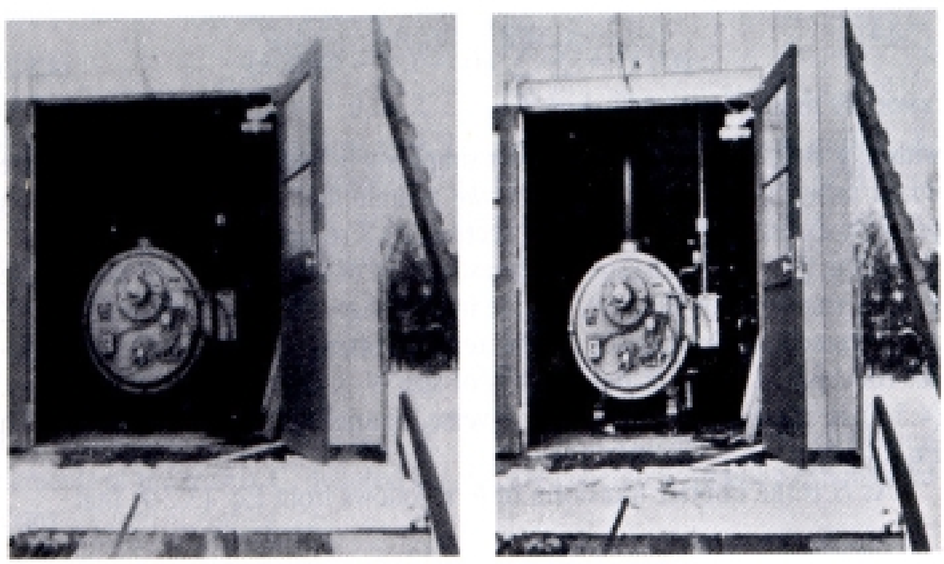


- Parameters ;

  $\gamma_L < 1, \gamma_H < 1$

- Filter ;
  - ✓ decrease the low freq.
  - ✓ amplify the high freq.
  - ⇒ simultaneous dynamic range compression and contrast enhancement

ex.)



(a) (b)

(a) original (b) $\gamma_L = 0.5 \quad \gamma_H = 1.5$

4.5 Generation of Spatial Masks from Frequency Domain Specification

- spatial domain processing :
  - simple to implement
  - short time to process

- given freq. domain filter :

  $\xrightarrow{appro.}$ spatial mask

  $$G(u,v) = H(u,v)F(u,v)$$

  $$g(x,y) = \sum_{i=0}^{N-1}\sum_{k=0}^{N-1} h(x-i, y-k) f(i,k)$$

  with x,y = 0,1,…,L-1

  h : spatial convolution mask

  $$H(u,v) = \frac{1}{N}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} h(x,y)\exp(-j2\pi(ux+vy)/N)$$

  for u,v = 0,1,…,L-1

  - suppose

    $h(x,y) = 0$ for $x > n$, $y > n$ with $n < N$

    → $n \times n$ convolution mask $\hat{h}$

    $$\hat{H}(u,v) = \frac{1}{N}\sum_{x=0}^{n-1}\sum_{y=0}^{n-1} \hat{h}(x,y)\exp(-j2\pi(ux+vy)/N)$$

    for u,v = 0,1,…,N-1

  - find $\hat{H}(u,v)$ with error minimization such as;

    $$e^2 = \sum_{u=0}^{N-1}\sum_{v=0}^{N-1} |\hat{H}(u,v) - H(u,v)|^2$$

  - in matrix form

    ✓ $\hat{\mathbf{H}} = \mathbf{C}\hat{\mathbf{h}}$

    where $\hat{\mathbf{H}}$: column vector of order N^2

    $\hat{\mathbf{h}}$: column vector of order n^2

    $\mathbf{C}$: $N^2 \times n^2$ matrix of exponential terms

- in matrix form ;

$$e^2 = (\hat{\mathbf{H}} - \mathbf{H})^* (\mathbf{H} - \hat{\mathbf{H}})$$

$$= \left\| \hat{\mathbf{H}} - \mathbf{H} \right\|^2$$

$$= \left\| \mathbf{C}\hat{\mathbf{h}} - \mathbf{H} \right\|^2$$

where ‖ ‖ : complex Euclidian norm.

- derivative ;

$$\frac{\partial e^2}{\partial \hat{\mathbf{h}}} = 2\mathbf{C}^* (\mathbf{C}\hat{\mathbf{h}} - \mathbf{H}) = 0$$

$$\rightarrow \hat{\mathbf{h}} = (\mathbf{C}^*\mathbf{C})^{-1} \mathbf{C}^* \mathbf{H}$$

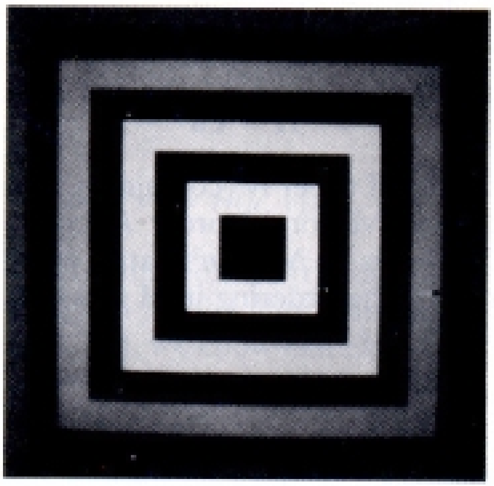
$$= \mathbf{C}^{\#} \mathbf{H}$$

Moore-Penrose generalized inverse

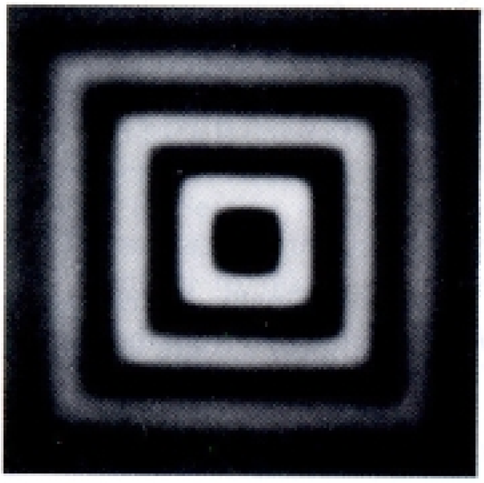
✓ specified N× N filter function h(x,y)

$$\xrightarrow[error]{\min.} n \times n \text{ convolution mask } \hat{h}(x, y)$$

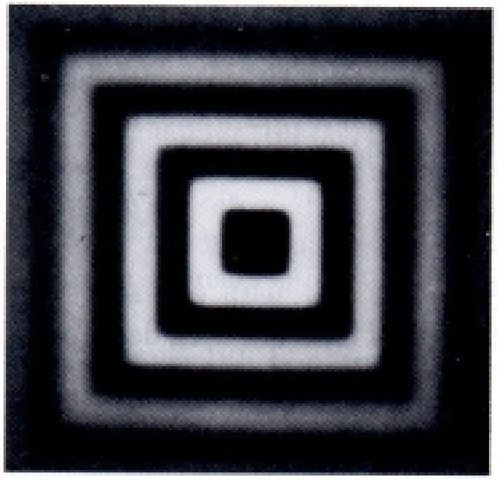
ex.)



(a)



(b)



(c)

b) filtered using a BLPF → blurred image

c) n=9

9× 9 convolution mask

filtered using spatial filter of 9× 9 convolution mask obtained by eg. (4.5-12)

: slightly less blurred than that obtained by using the complete filter in freq. Domain