Implementation

- Finding $\|n\|$ for x=0, 1, 2, 3, …., M-1, and y=0, 1, 2, 3, ….., N-1

$$\begin{aligned}\sigma_\eta^2 &= E\{[\eta_e(x, y) - \bar{\eta}_e]^2\} \\ &= E[\eta_e^2(x, y)] - \bar{\eta}_e^2\end{aligned}$$

$$\bar{\eta}_e = \frac{1}{(M-1)(N-1)} \sum_x \sum_y \eta_e(x, y)$$

$$\sigma_\eta^2 = \frac{1}{(M-1)(N-1)} \sum_x \sum_y \eta_e^2(x, y) - \bar{\eta}_e^2.$$

$$\sigma_\eta^2 = \frac{\|\mathbf{n}\|^2}{(M-1)(N-1)} - \bar{\eta}_e^2$$

$$\|\mathbf{n}\|^2 = (M-1)(N-1)[\sigma_\eta^2 + \bar{\eta}_e^2].$$

  - $\|n\|$ can be approximated or measured.

- Constraint least squares restoration procedure

Step 1. Choose an initial value of γ and obtain an estimate of $\|\mathbf{n}\|^2$ by using Eq. (5.6-27).

Step 2. Compute $\hat{F}(u, v)$ using Eq. (5.6-18). Obtain $\hat{\mathbf{f}}$ by taking the inverse Fourier transform of $\hat{F}(u, v)$.

Step 3. Form the residual vector $\mathbf{r}$ according to Eq. (5.6-19) and compute $\phi(\gamma) = \|\mathbf{r}\|^2$.

Step 4. Increment or decrement γ.

(a) $\phi(\gamma) < \|\mathbf{n}\|^2 - a$. Increment γ according to the algorithm given above or other appropriate method (such as a Newton–Raphson procedure).

(b) $\phi(\gamma) > \|\mathbf{n}\|^2 + a$. Decrement γ according to an appropriate algorithm.

Step 5. Return to step 2 and continue unless step 6 is true.

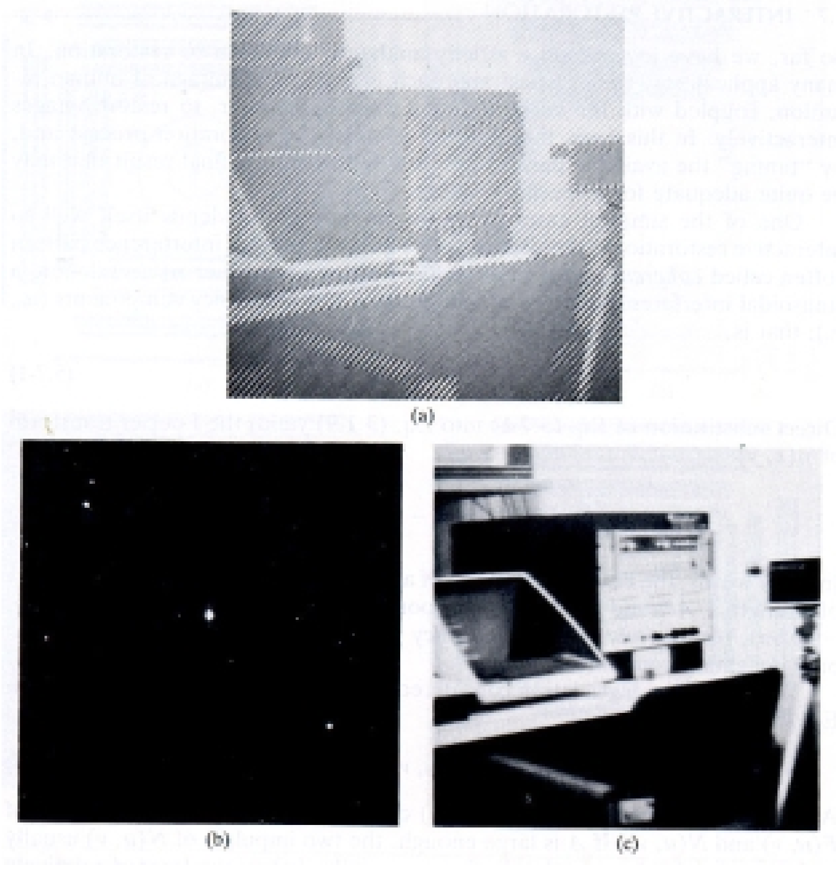
Step 6. $\phi(\gamma) = \|\mathbf{n}\|^2 \pm a$, where a determines the accuracy with which the constraint is satisfied. Stop the estimation procedure, with $\hat{\mathbf{f}}$ for the present value of γ being the restored image.

5.7 Interactive Restoration

- Observer controls the restoration process by tuning the available parameters
  To obtain a final result that may be quite adequate for a specific result

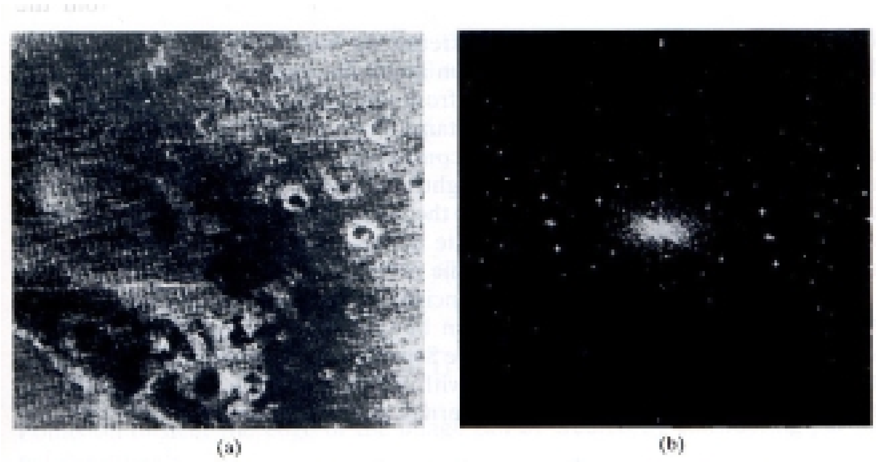
- Coherent noise : 2D sinusoidal interference pattern
  $\eta(x, y) = A\sin(u_0 x + v_0 y)$
  - FT
    $$N(u,v) = \frac{-jA}{2}\left[\delta\left(u - \frac{u_0}{2\pi}, v - \frac{v_0}{2\pi}\right) - \delta\left(u + \frac{u_0}{2\pi}, v + \frac{v_0}{2\pi}\right)\right]$$
  - Additive noise
    $G(u,v) = F(u,v) + N(u,v)$
    $N(u,v)$: appear as bright dots in freq. Domain

ex.)



use of Band Reject Filter to reject sinusoidal pattern

- periodic image degradation : commonly derived from electro-optical scanner (used in space mission)
  - ex.)



(a) composed of more than just one sinusoidal component

(b) FT of (a)

  - filtering : may remove too much image inform
  - procedure
    i. isolating the principal contribution of interference scene
    ii. subtracting the weighted pattern from the corrupted image

- the first step
  - extract the principal frequency components of interference pattern
  - by placing a BPF H(u,v) at the location of each spike
  - FT of pattern

$P(u,v) = H(u,v)G(u,v)$

FT of corrupted image g(x,y)

for u,v = 0, 1, 2, …, N-1

  - BPF H(u,v)
    - ✓ Constructed to pass only components associated with the interference pattern
    - ✓ Constructed interactively by observing the spectrum of G(u,v)
  - Corresponding pattern

$p(x,y) = F^{-1}\{H(u,v)G(u,v)\}$: approximation of true pattern

- $\hat{f}(x,y) = g(x,y) - w(x,y)p(x,y)$

  where $\hat{f}(x,y)$: estimate of f(x,y)

  $w(x,y)$: weighting function

- $w(x,y)$

: selected so that the variance of $\hat{f}(x,y)$ is minimized over a specified neighborhood of every point (x,y)

- neighborhood : size $(2X+1)$ by $(2Y+1)$ about (x,y)
- local variance of $\hat{f}(x,y)$ at (x,y)

$$\sigma^2(x,y) = \frac{1}{(2X+1)(2Y+1)} \sum_{m=-X}^{X} \sum_{n=-Y}^{Y} \left[\hat{f}(x+m,y+n) - \bar{\hat{f}}(x,y) \right]^2$$

$\bar{\hat{f}}$: average of $\hat{f}$ in neighborhood

that is $\bar{\hat{f}}(x,y) = \frac{1}{(2X+1)(2Y+1)} \sum_{m=-X}^{X} \sum_{n=-Y}^{Y} \hat{f}(x+m,y+n)$

With the relationship of $\hat{f}(x,y) = g(x,y) - w(x,y)p(x,y)$

$$\sigma^2(x,y) = \frac{1}{(2X+1)(2Y+1)} \sum_{m=-X}^{X} \sum_{n=-Y}^{Y} \{[g(x+m,y+n)$$

$$-w(x+m,y+n)p(x+m,y+n)] - [\bar{g}(x,y) - \overline{w(x,y)p(x,y)}]\}^2$$

- let us assume

$w(x,y)$: constant over neighborhood

$w(x+m,y+n) = w(x,y)$ for $-X \le m \le X$, $-Y \le m \le Y$

$\rightarrow \overline{w(x,y)p(x,y)} = w(x,y)\bar{p}(x,y)$

Finally

$$\sigma^2(x,y) = \frac{1}{(2X+1)(2Y+1)} \sum_{m=-X}^{X} \sum_{n=-Y}^{Y} \{[g(x+m, y+n)$$
$$- w(x,y)p(x+m, y+n)] - [\overline{g}(x,y) - w(x,y)\overline{p}(x,y)]\}^2$$

to minimize $\sigma^2(x,y)$

$$\frac{\partial \sigma^2(x,y)}{\partial w(x,y)} = 0 \qquad \text{for} \quad w(x,y)$$

$$w(x,y) = \frac{\overline{g(x,y)p(x,y)} - \overline{g}(x,y)\overline{p}(x,y)}{\overline{p^2}(x,y) - \overline{p}^2(x,y)}$$

: computed for one point in each nonoverlapping neighborhood (preferably center point)

ex.) fig. 5.9 : FT of fig. 5.8 (a) (without shifting)

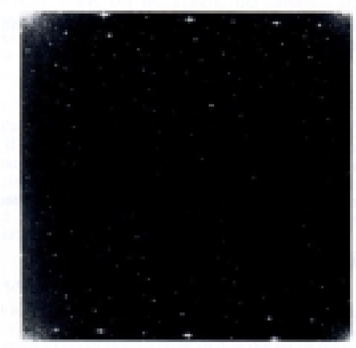
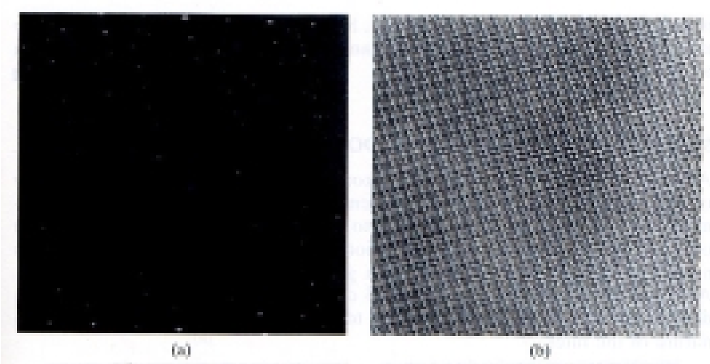


fig. 5.10



(a)FT of p(u,v)

(b) Corresponding interference pattern p(x,y0

fig .5.11 : processed image