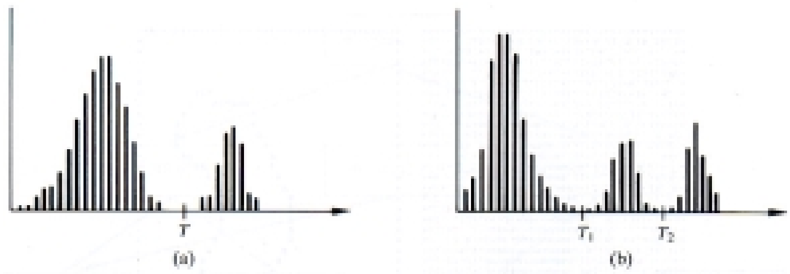
7.3 Thresholding

- the most important approaches to image segmentation

7.3.1 Foundation



(a)

- light objects on a dark background
- grouped into two dominant modes
- select threshold T that separates these modes
- object point : $f(x, y) > T$

  background point : otherwise

(b)

- three dominant mode

  ex. 2 types of light objects

  a dark background

- $T_1 < f(x, y) \le T_2$ — Two object

  $f(x, y) > T_2$ — Two object

  $f(x, y) \le T_1$ — background

- multilevel thresholding

  : generally less reliable than single thresholding

  : difficulty of establishing multiple thresholdings that effectively isolate regions

- thresholding operation

  $$T = T[x, y, p(x, y), f(x, y)]$$

  where $f(x, y)$: gray level of point (x, y)

  $p(x, y)$: some local property of this point

  ex.) average gray level of neighborhood centered on (x, y)

- thresholded image

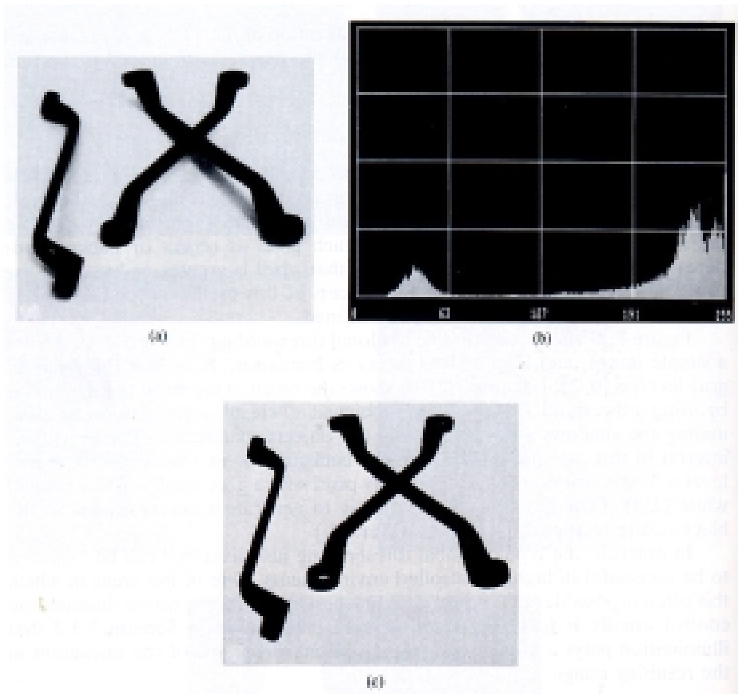
$$g(x,y)=\begin{cases}1 & if\ f(x,y)>T \quad :object \\ 0 & if\ f(x,y)\le T \quad :backgound\end{cases}$$

if T : depend only on $f(x,y)$ $\rightarrow$ global

T : depend on both $f(x,y)$ and $p(x,y) \rightarrow$ local

T : depend on the spatial coordi. x angle $\rightarrow$ dynamic

7.3.3 Simple Global Thresholding



- threshold T=70
- eliminating shadow
- any pixels with gray level $\le T$ $\rightarrow$ black (0)

  any pixels with gray level $> T$ $\rightarrow$ white (255)

  $\rightarrow$ binary image (c)
- this method : successful in highly controlled environment

  ex.) industrial inspection applications

  where illumination control : usually feasible

  (Θ illumination : a crucial role in establishing the shape of the histogram in resulting image)

7.3.4 Optimal Thresholding

- only two principal brightness regions
  - histogram : estimate of brightness probability density function p(z)
- ✓ sum or mixture of two unimodel densities : one for the light and one for the dark regions
- ✓ mixture parameter : proportional to the area of picture of each brightness
  - if form of densities : known → segmentation : possible
- mixture probability density function
  - $p(z) = P_1 p_1(z) + P_2 p_2(z)$

    for the Gaussian case

$$p(z) = \frac{P_1}{\sqrt{2\pi}\sigma_1}\exp\left[-\frac{(z-\mu_1)^2}{2\sigma_1^2}\right] + \frac{P_2}{\sqrt{2\pi}\sigma_2}\exp\left[-\frac{(z-\mu_2)^2}{2\sigma_2^2}\right]$$

    where μ_1, μ_2 : mean
- ✓ constraint $P_1 + P_2 = 1$
- ✓ five unknown parameters
  - if all parameters : known

    → optimal threshold : easily determined
- suppose
  - dark region : background
  - bright region : object
  - $\mu_1 < \mu_2$ threshold T
  - gray level $\le T$ → background points

    gray level $> T$ → object points
  - error prob. (object → background points)

$$E_1(T) = \int_{-\infty}^{T} p_2(z)dz$$

    background → object points error prob.

$$E_2(T) = \int_{T}^{\infty} p_1(z)dz$$
  - overall prob. of error

$$E(T) = P_2 E_1(T) + P_1 E_2(T)$$
  - min. error
    - ✓ differentiating with T and equaling result to 0
    - ✓ $P_1 p_1(T) = P_2 p_2(T)$

      → $AT^2 + BT + C = 0$

$$A = \sigma_1^2 - \sigma_2^2$$

$$B = 2(\mu_1\sigma_2^2 - \mu_2\sigma_1^2)$$

$$C = \sigma_1^2\mu_2^2 + \sigma_2^2\mu_1^2 + 2\sigma_1^2\sigma_2^2 \ln(\sigma_2 P_1 / \sigma_1 P_2)$$

- if variances : equal, $\sigma_1^2 = \sigma_2^2 = \sigma^2$

  → single threshold

$$T = \frac{\mu_1 + \mu_2}{2} + \frac{\sigma^2}{\mu_1 - \mu_2} \ln\left(\frac{P_2}{P_1}\right)$$

- if prior probs. Are equal, $P_1 = P_2 = P$

  → single threshold

$$T = (\mu_1 + \mu_2)/2$$

7.3.5 Threshold Selection Based on Boundary Characteristics

- selection of good threshold
  - histogram peaks : tall, narrow, symmetric, separated by deep valleys
  - consider only those pixels that lie on or near the boundary between objects and background

    → improve the symmetry of the histogram peaks
  - using pixels that satisfy some simple measures based on gradient and Laplacian operators

    → tendency to deepen the valley between histogram peaks
  - valleys of histogram formed from the pixels selected by a gradient/Laplacian criterion : sparsely populated

    → highly desirable deep valleys
- gradient ∇f (7.1-4) (7.1-5)

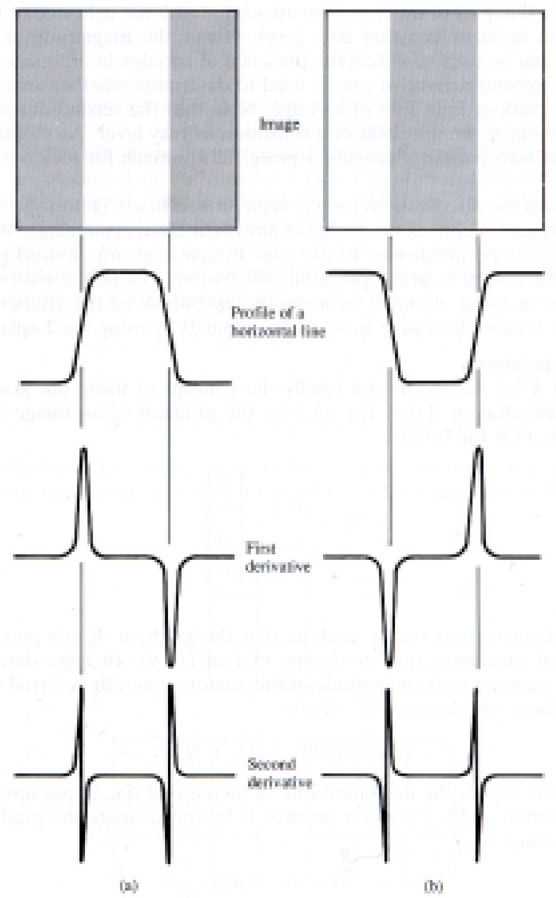
  Laplacian $\nabla^2 f$ (7.1-10)

  → three level image

$$s(x,y) = \begin{cases} 0 & if \quad \nabla f < T & \text{: homogeneous region} \\ + & if \quad \nabla f \geq T \quad and \quad \nabla^2 f \geq 0 & \rightarrow \text{dark} \\ - & if \quad \nabla f \geq T \quad and \quad \nabla^2 f < 0 & \rightarrow \text{bright} \end{cases}$$

  : three distinct gray levels

  +, − : reversed for a light object on a dark background



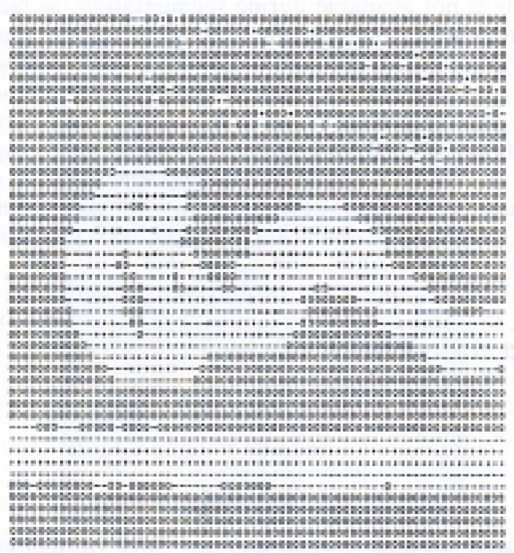
- For example, object → 1 background → 0

  transition

  from light background to dark object → (−, +)

  from object to background → (+, −)

  interior of object → 0 or +

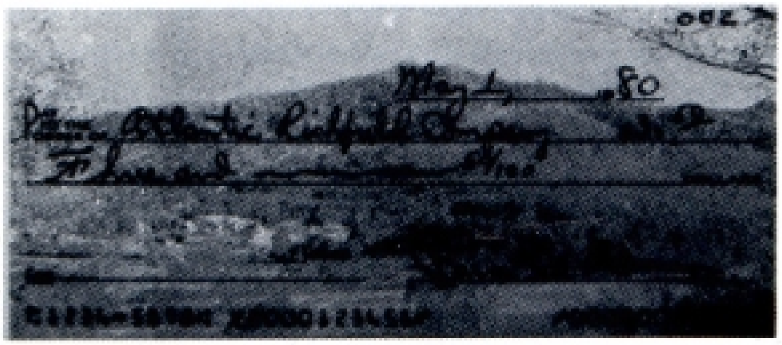


- horizontal or vertical scan line containing a section of an object

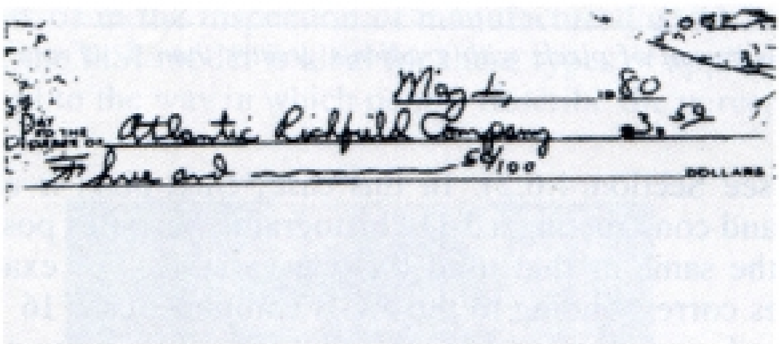
$(\Lambda)(-,+)(0\ or\ +)(+,-)(\Lambda)$

b→0   Interior of 0   0→b

ex.)

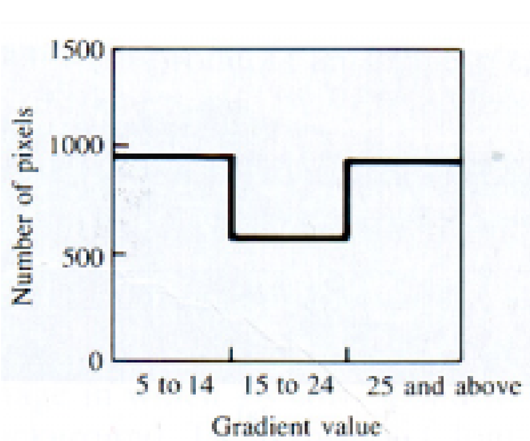


(a)



(b)

- histogram : two dominant modes which are symmetric, same height and are separated by a distinct valley
- T : midpoint of valley



- histogram as a function of gradient value for pixels with gradients greater than 5

7.3.6 Thresholds based on Several Variables

- so far, thresholding a single intensity variable
- more than one variable
  - characterize each pixel in an image

  ex.) color image (RGB components)
    - ✓ 3-D histogram
    - ✓ processing : the same as that used for 1-D processing (one variable)
    - ✓

  ex.) for three 16-level color image (RGB)

    → 16×16 ×16grid (cube) : formed

    → the no. of pixels : inserted in each cell : 3-D histogram

    division → normalized histogram
    - ✓ thresholding : find clusters of points in 3-D space
    - ✓ one cluster $\xrightarrow{assign}$ one intensity

      other cluster $\xrightarrow{assign}$ other intensity
    - ✓ seeking cluster : increasingly complex task as the no. of variable increase

ex.)

- HIS model
  - hue, saturation → color components
  - automated inspection

    ex.) inspection of fruits in ripeness

    inspection of manufactured goods
  - closely related to human perception of color